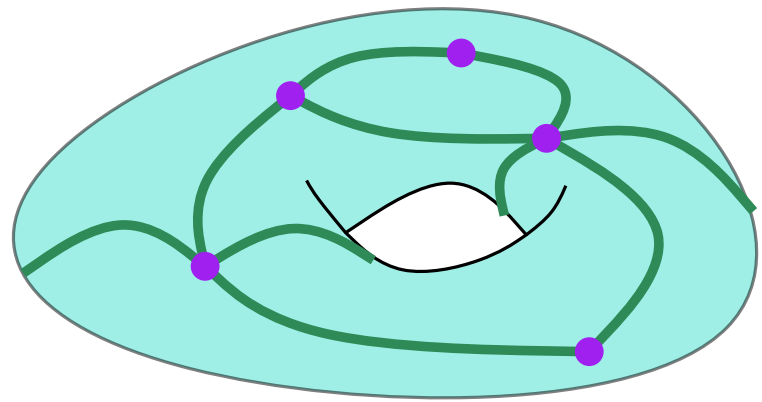
Straight line drawing of a graph on the flat torus

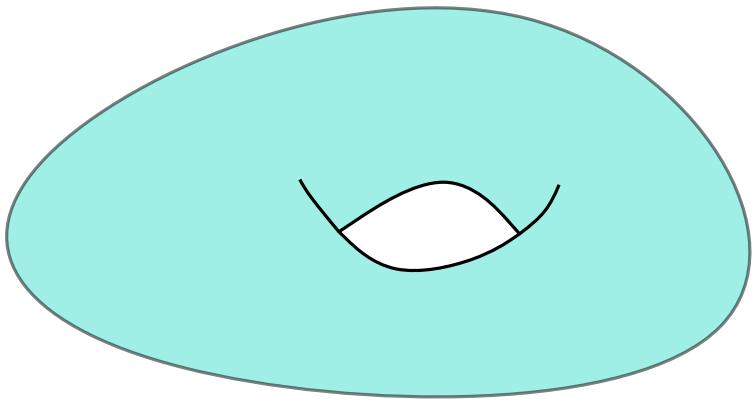
Luca Castelli Aleardi, LIX

Olivier Devillers, INRIA

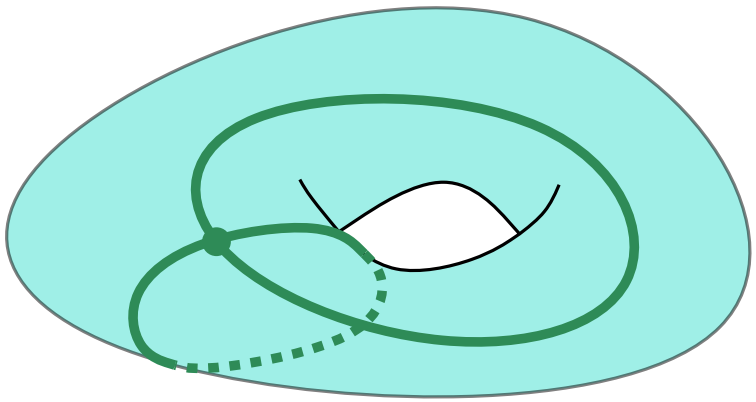
Éric Fusy, LIX



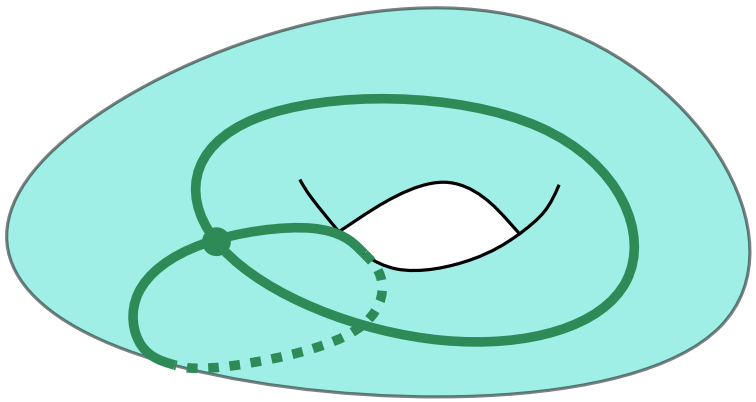
Torus



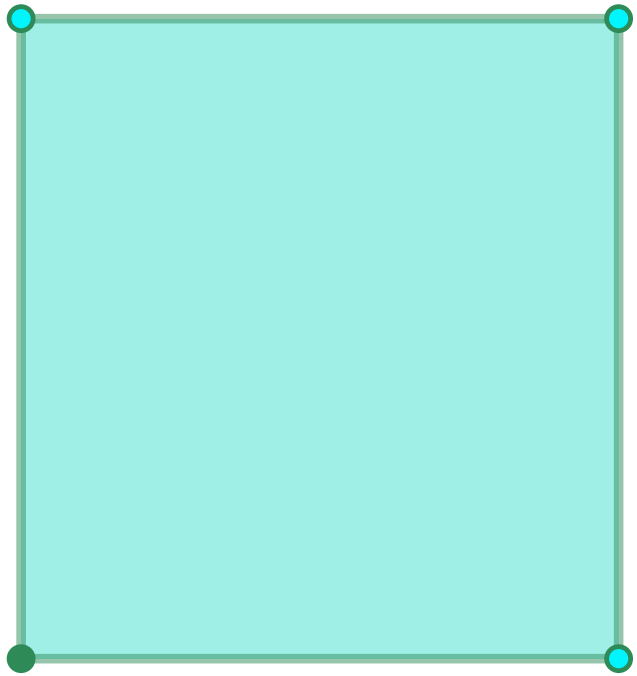
Torus



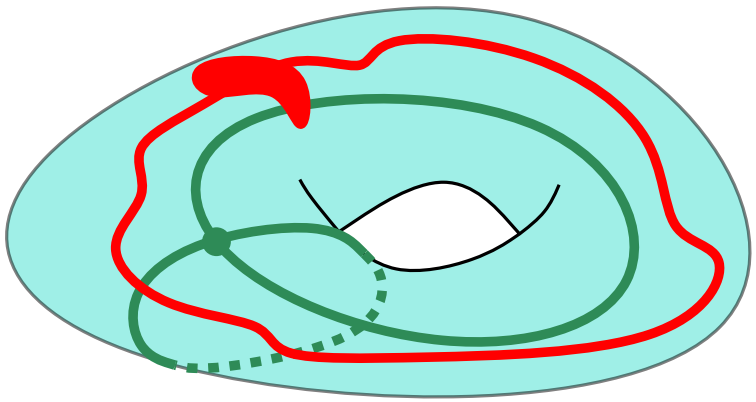
Torus



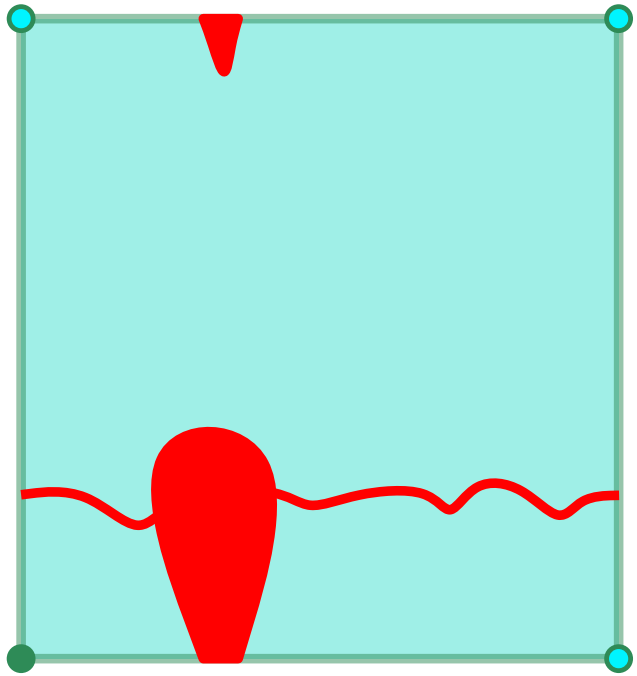
Flat torus



Torus

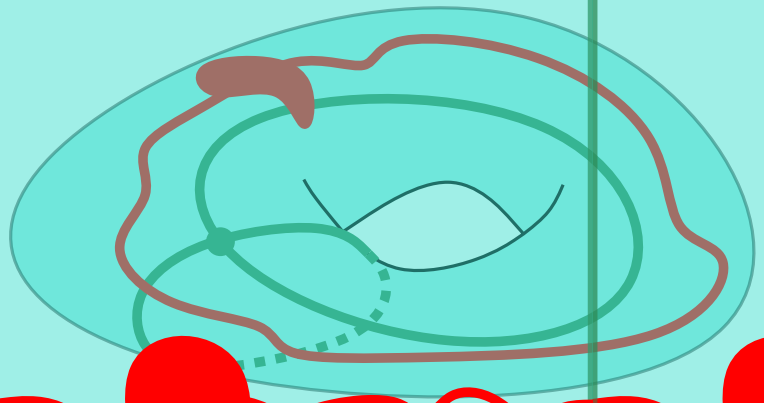


Flat torus

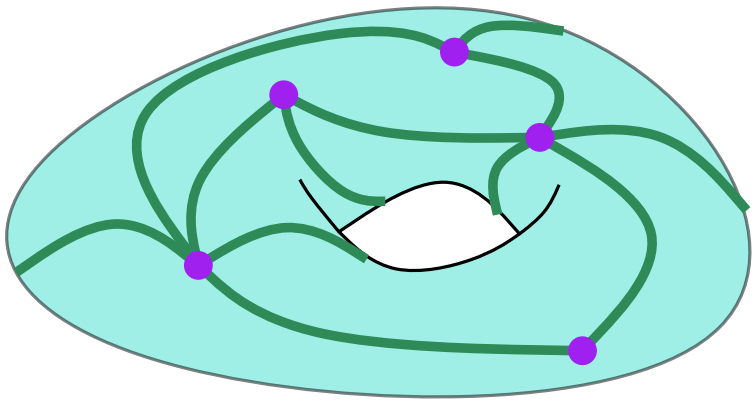


Torus

Flat torus

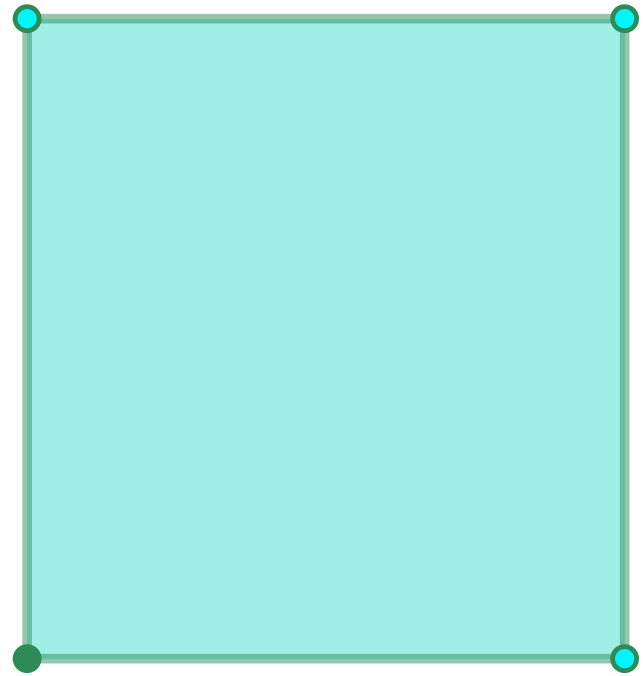


Torus



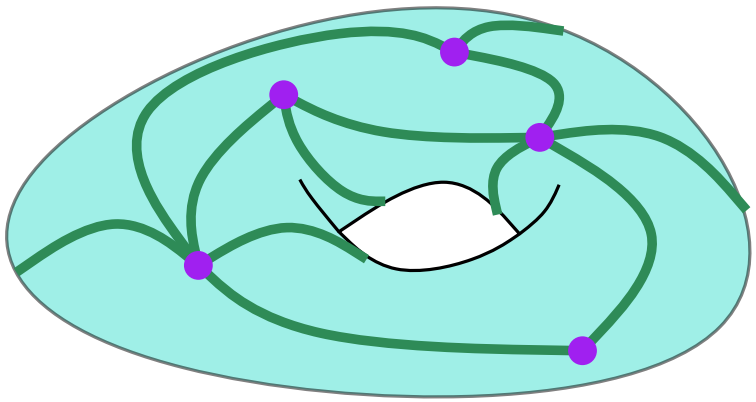
Graph

Flat torus



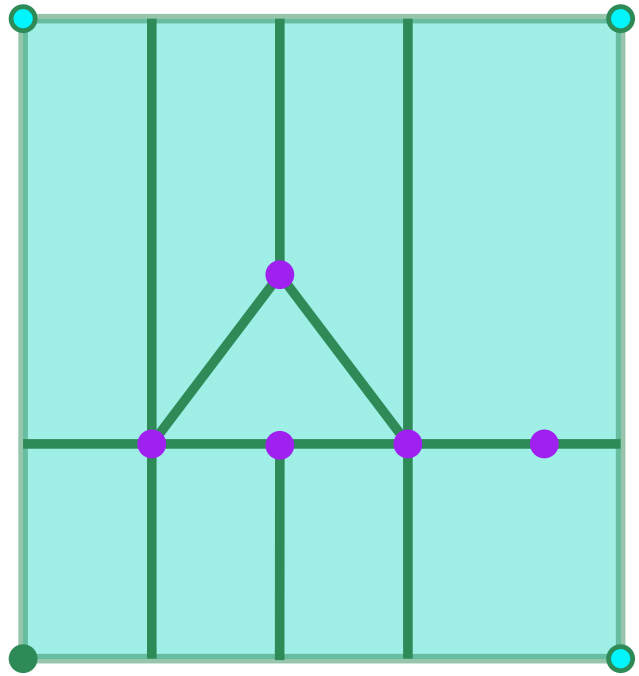
Problem statement

Torus



Graph

Flat torus



Convex straight line drawing

Result

Given a map on a torus

Result

Given a map on a torus
(essentially 3-connected)

Get a

Convex straight line drawing
in rectangle $O(n) \times O(n^{\frac{3}{2}})$

[De Fraysseix, Pach, & Pollack]

Given a triangulation

Get a

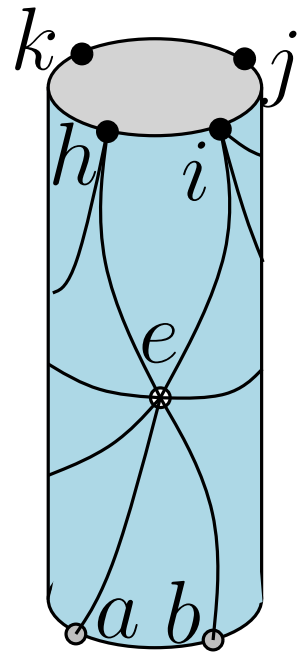
Planar straight line drawing in rectangle $O(n) \times O(n)$

Algorithm, global view

- triangulation of a cylinder, no chords, no 2-cycles
  - Vertices ordering
  - Incremental drawing (boudary characteristics)
- triangulation of a cylinder, with chords, 2-cycles & loops
  - Split triangulation in pieces
- maps of a cylinder
  - Different boudary characteristics
- triangulation and maps of a torus
  - Cut the torus $\rightarrow$ cylinder

Algorithm, vertices ordering

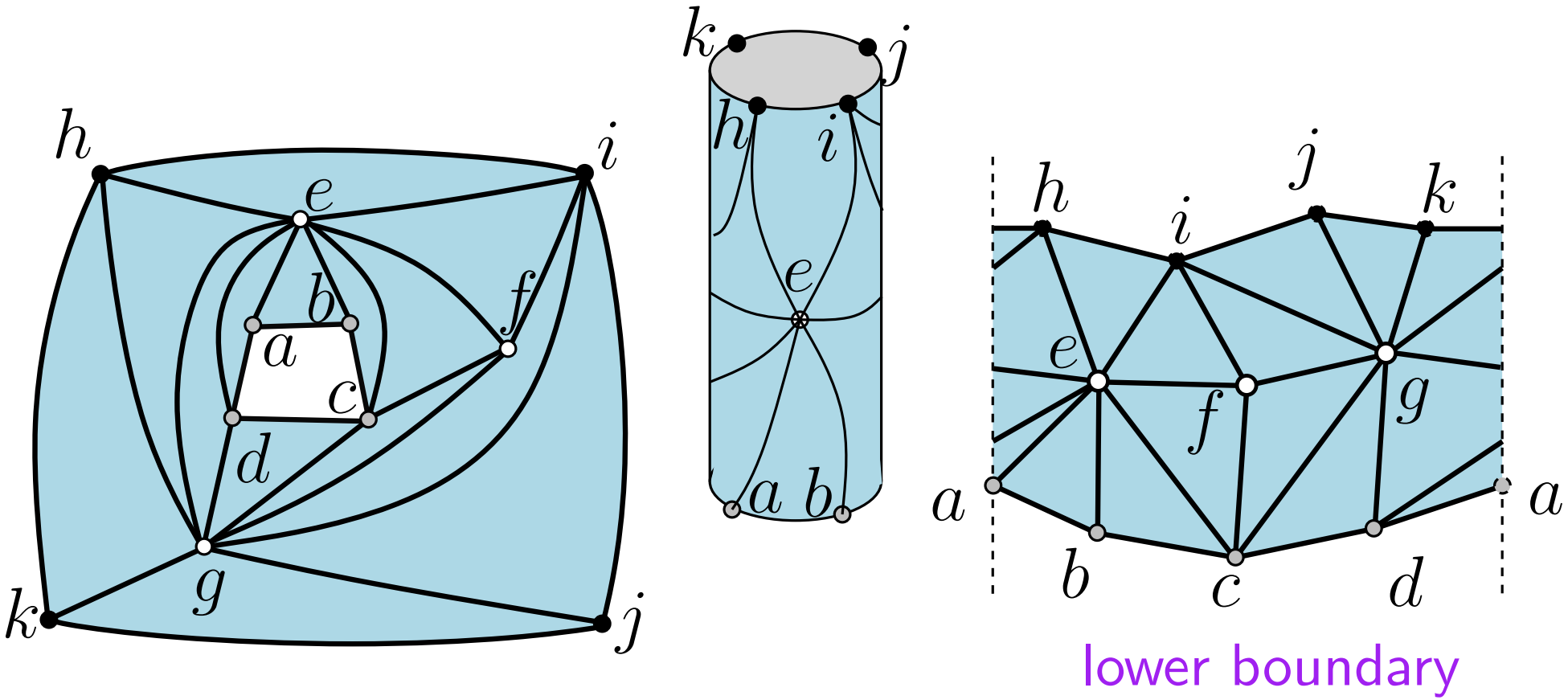
triangulation of a cylinder, no chords, no 2-cycles



Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles

Annular view / periodic view

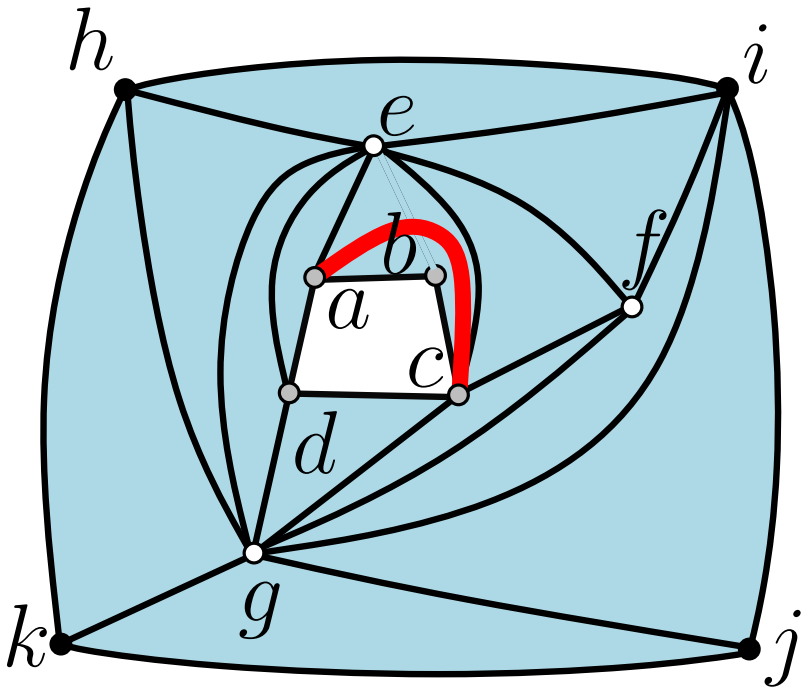


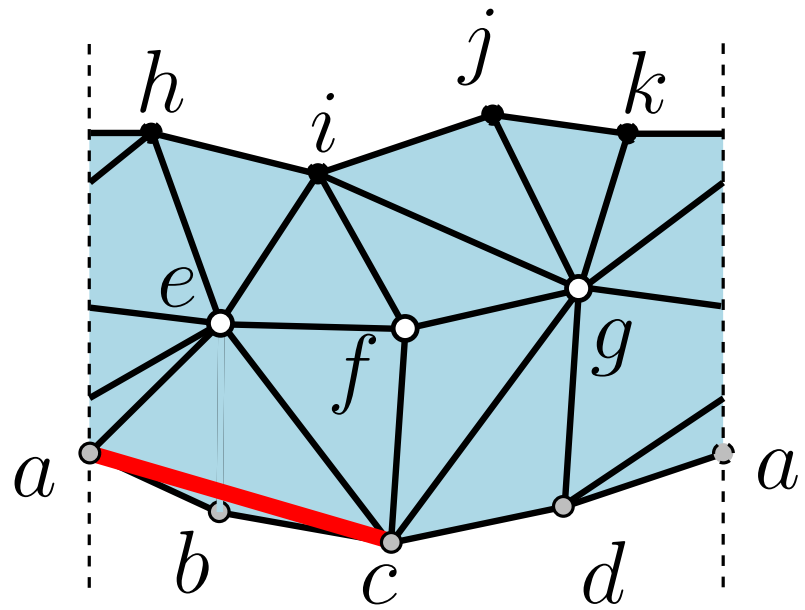
lower boundary

Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles

Annular view / periodic view





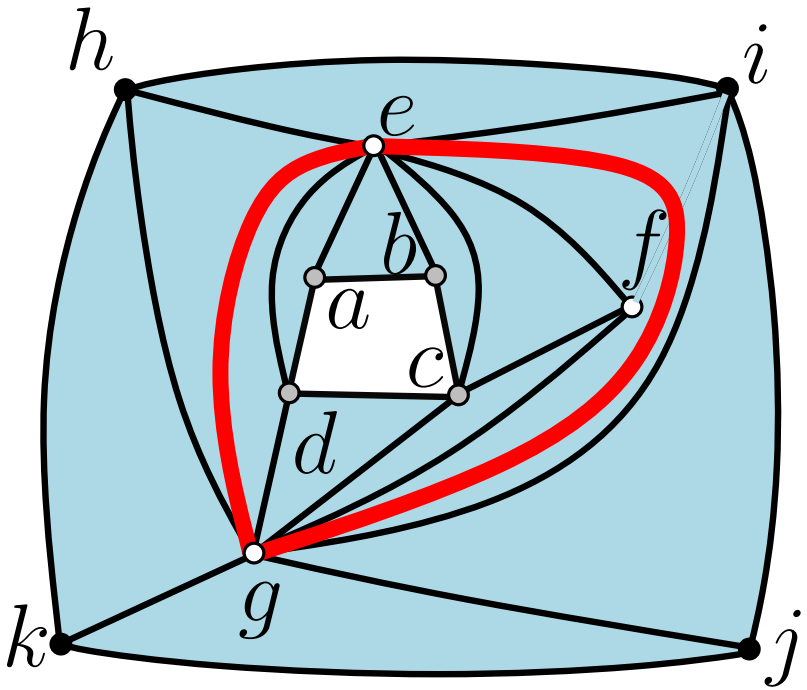
lower boundary

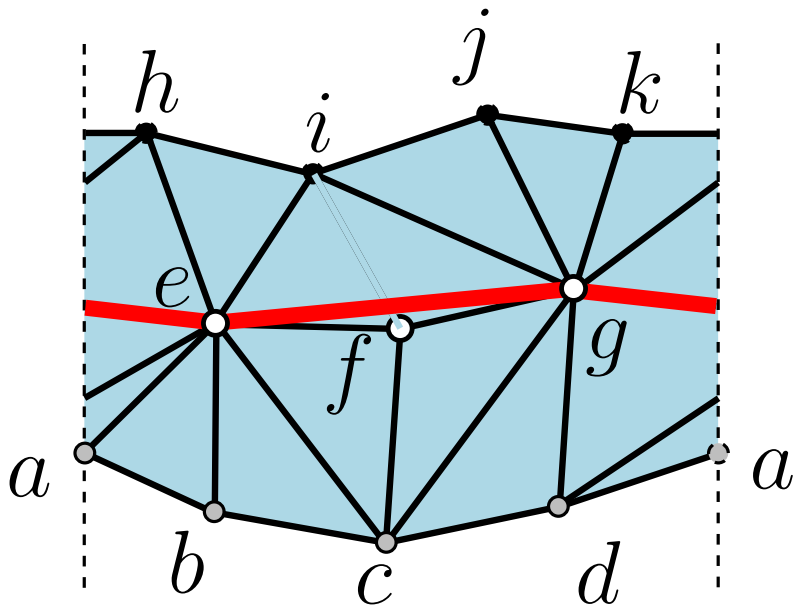
no chords at lower boundary

Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles

Annular view / periodic view





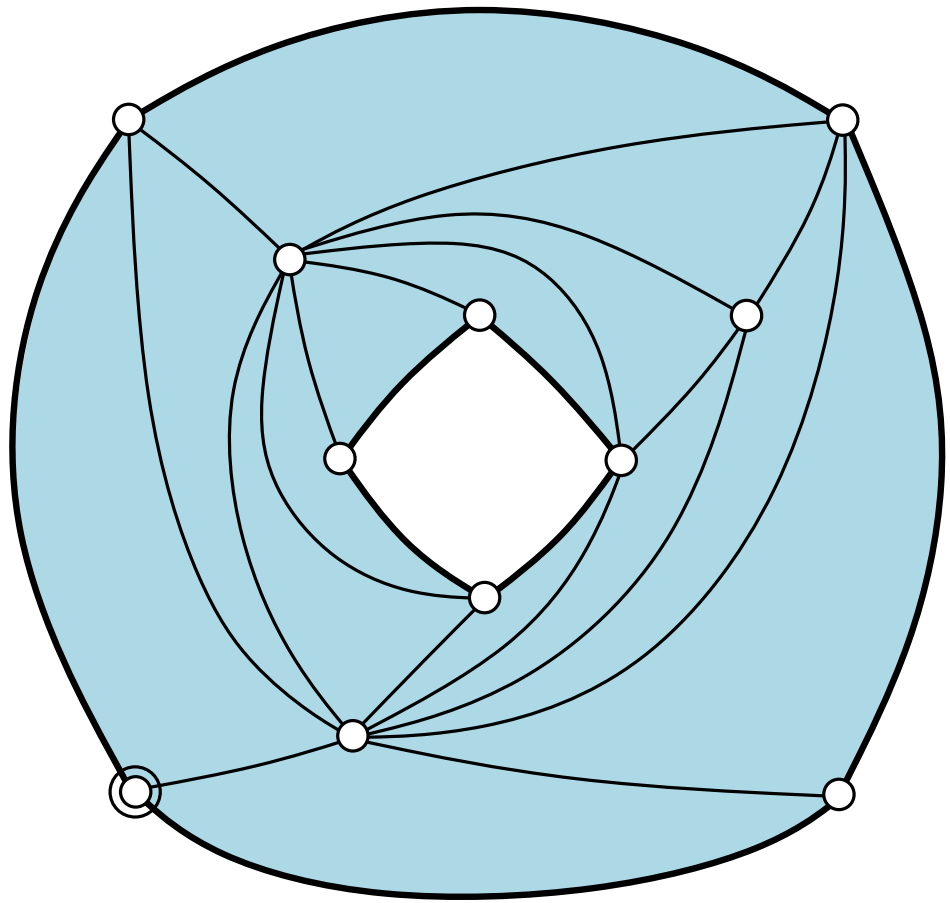
lower boundary

no 2-cycles

Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles

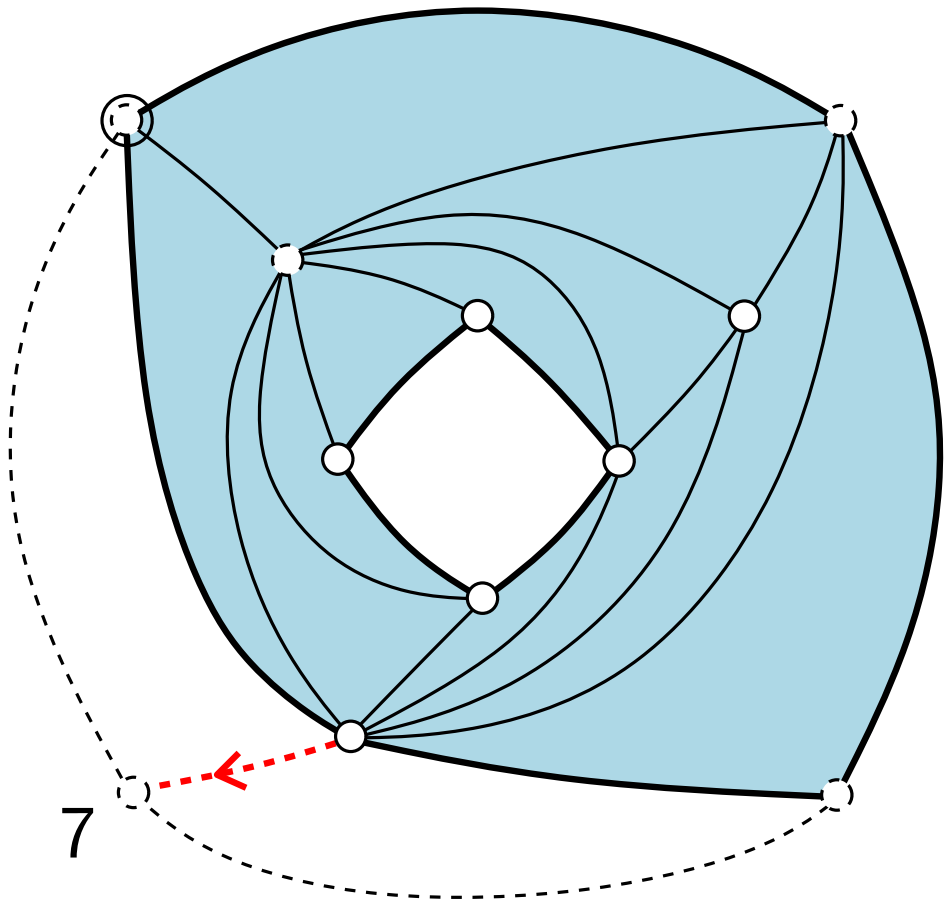
Shelling order



Algorithm, vertices ordering

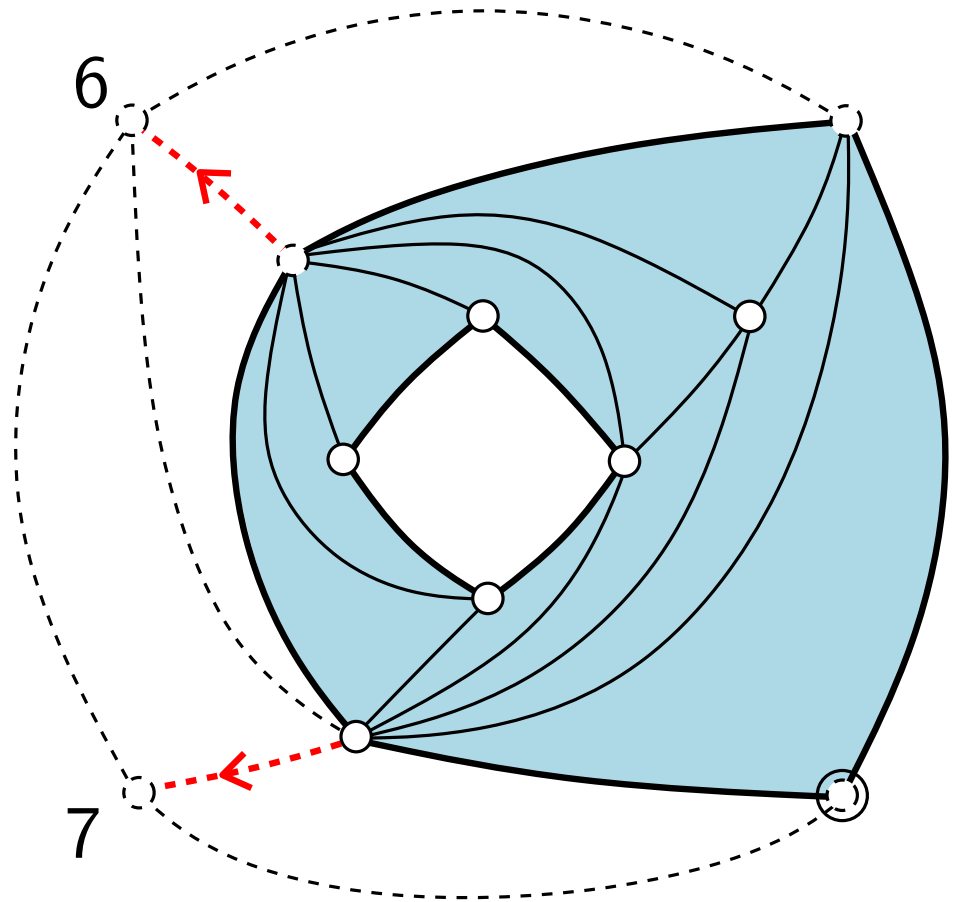
triangulation of a cylinder, no chords, no 2-cycles

Remove vertices, so that the remaining part is an annulus



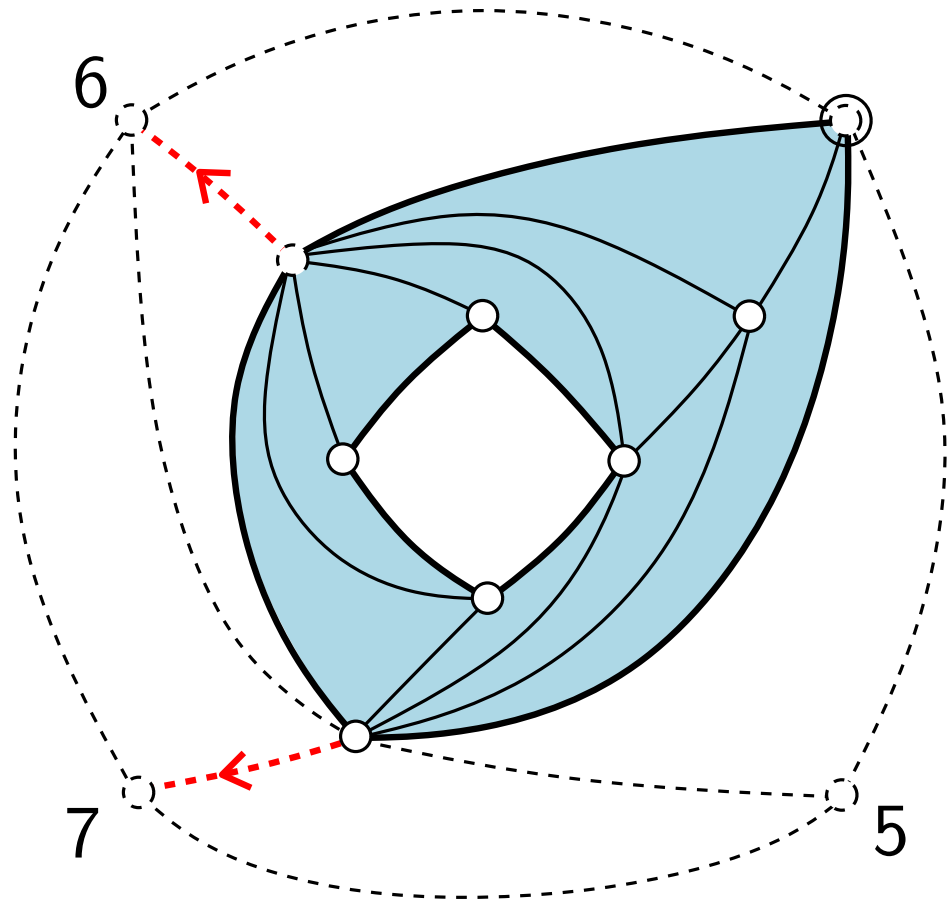
Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles



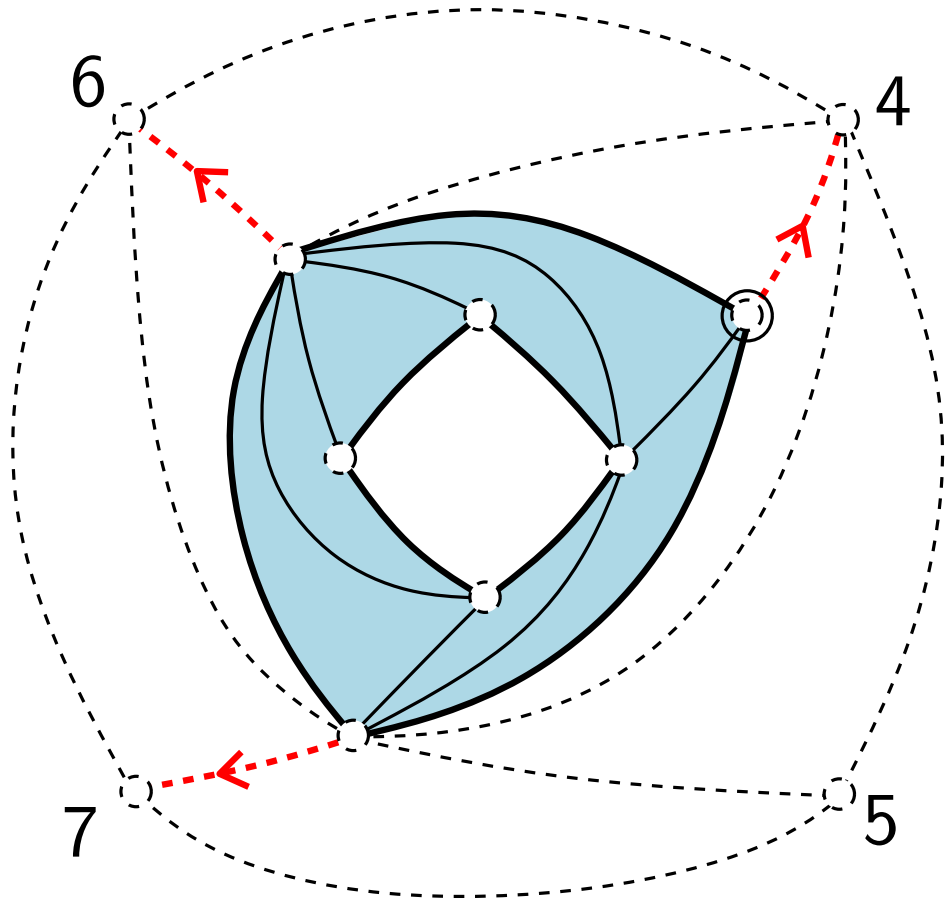
Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles



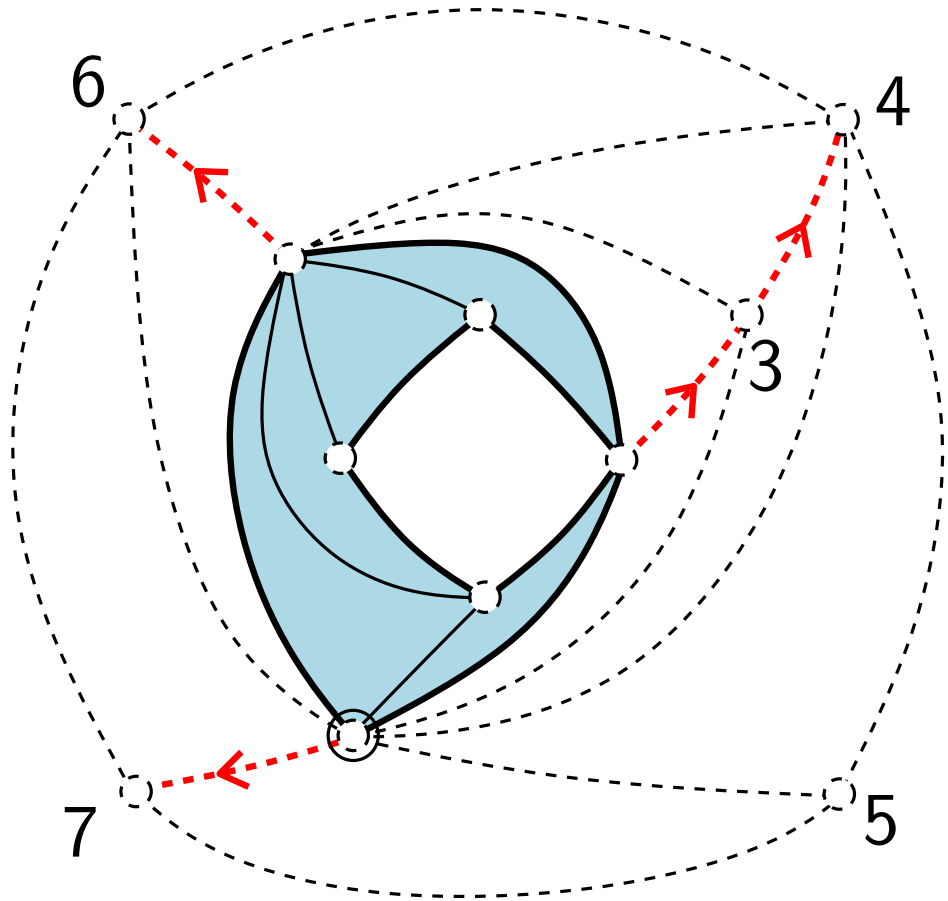
Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles



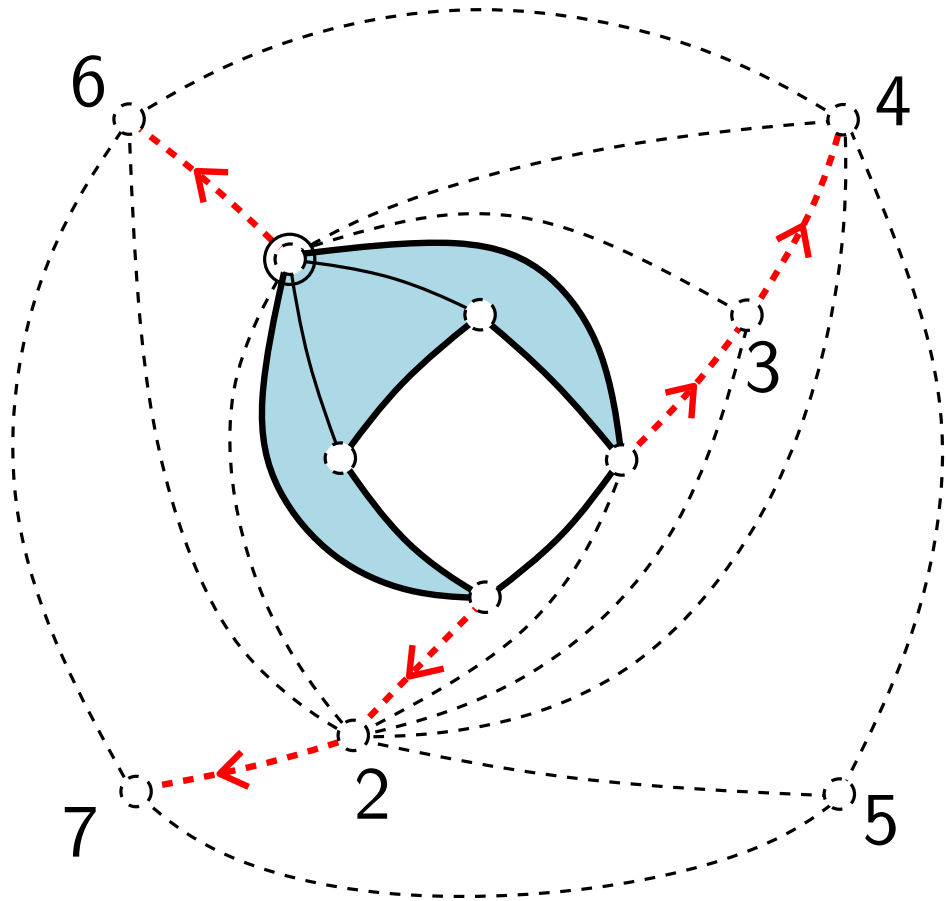
Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles



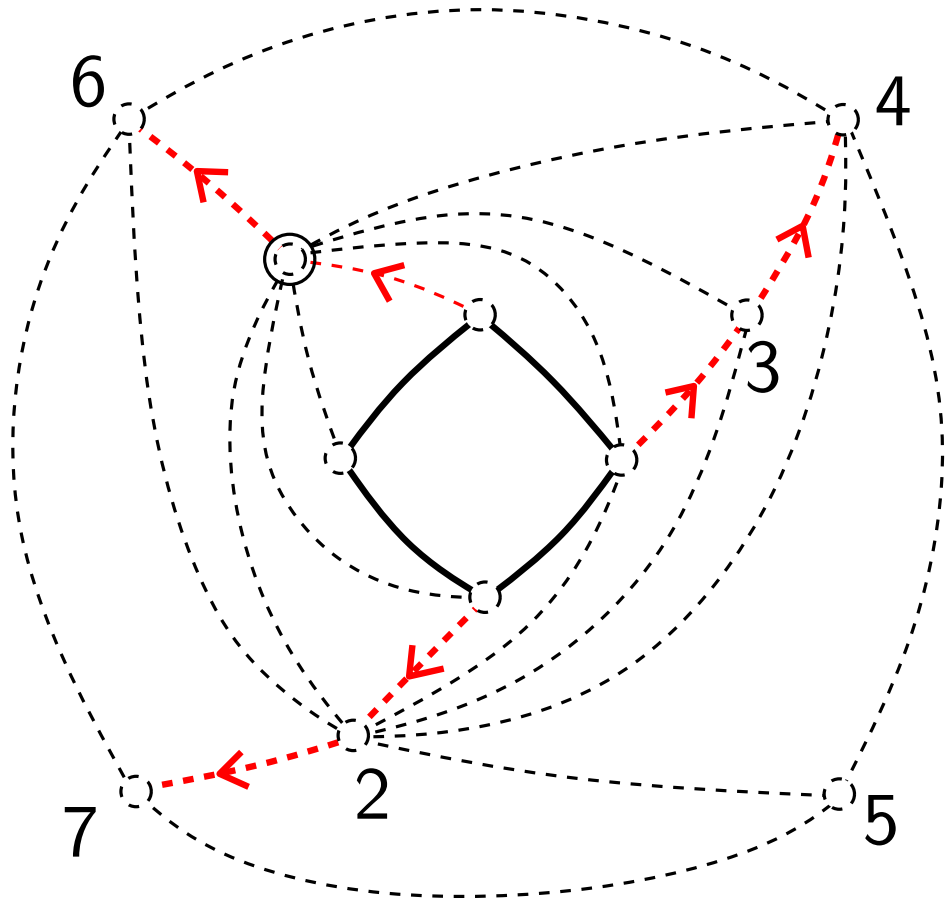
Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles



Algorithm, vertices ordering

triangulation of a cylinder, no chords, no 2-cycles



Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

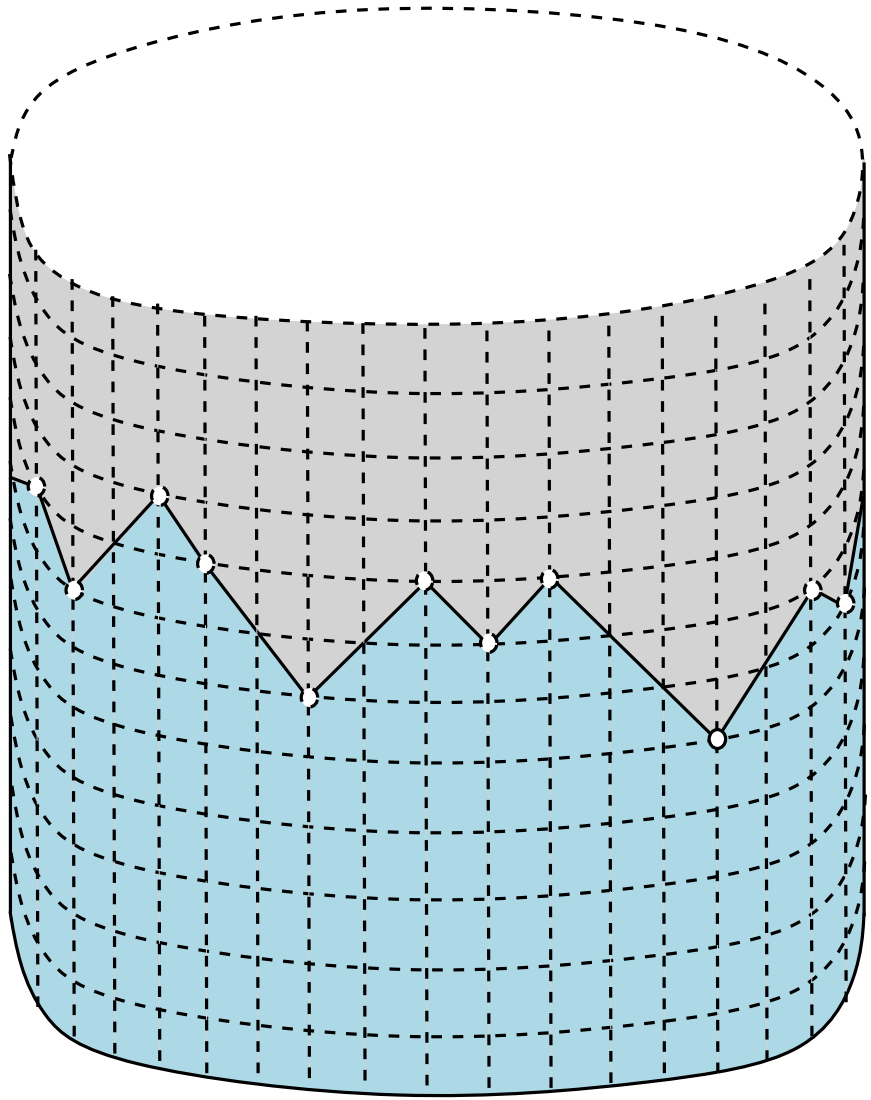
incremental drawing

Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

incremental drawing

slopes +1 or -1

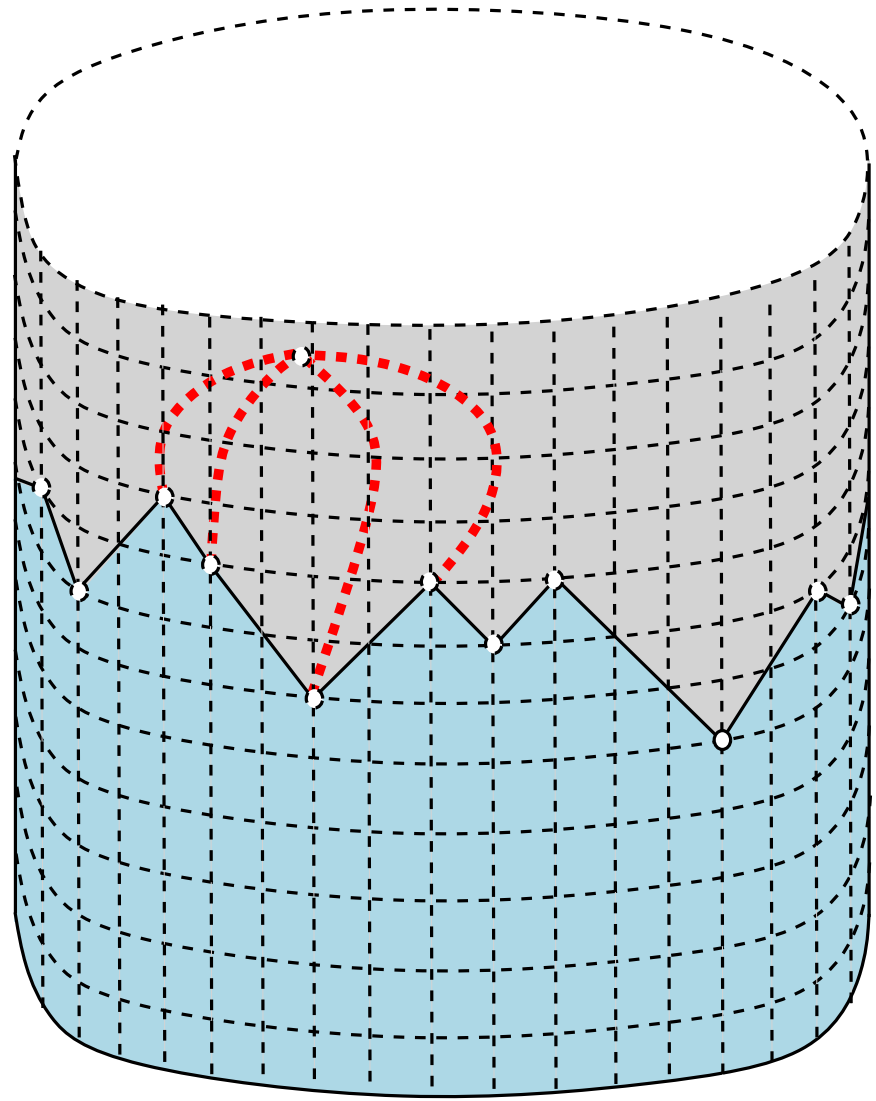


Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

incremental drawing

slopes +1 or -1

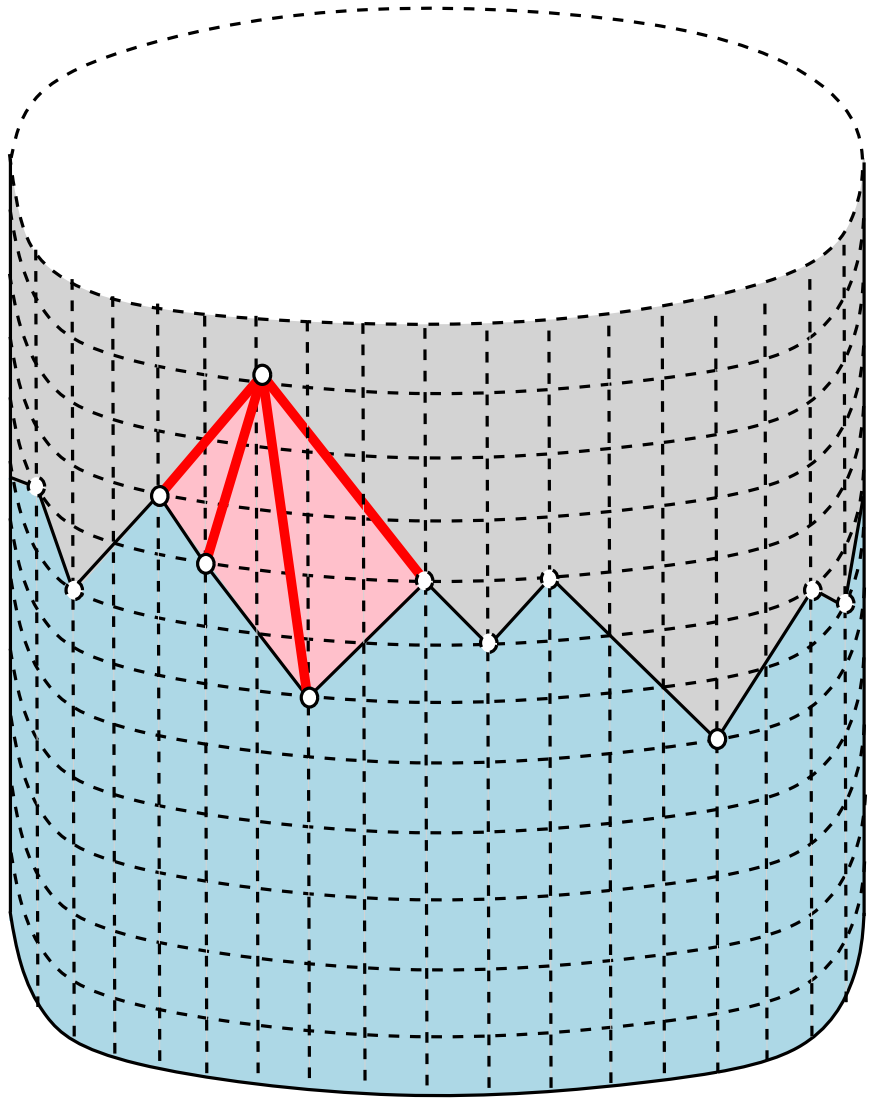


Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

incremental drawing

slopes +1 or -1

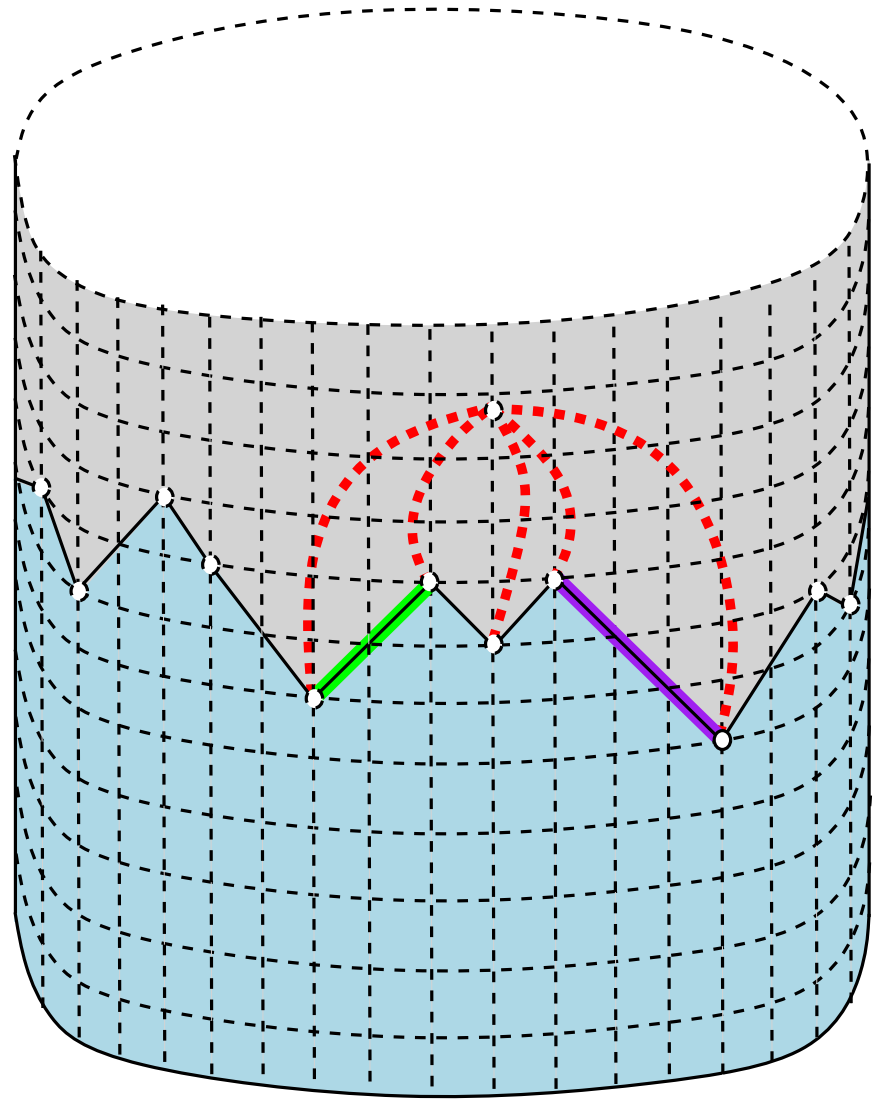


Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

incremental drawing

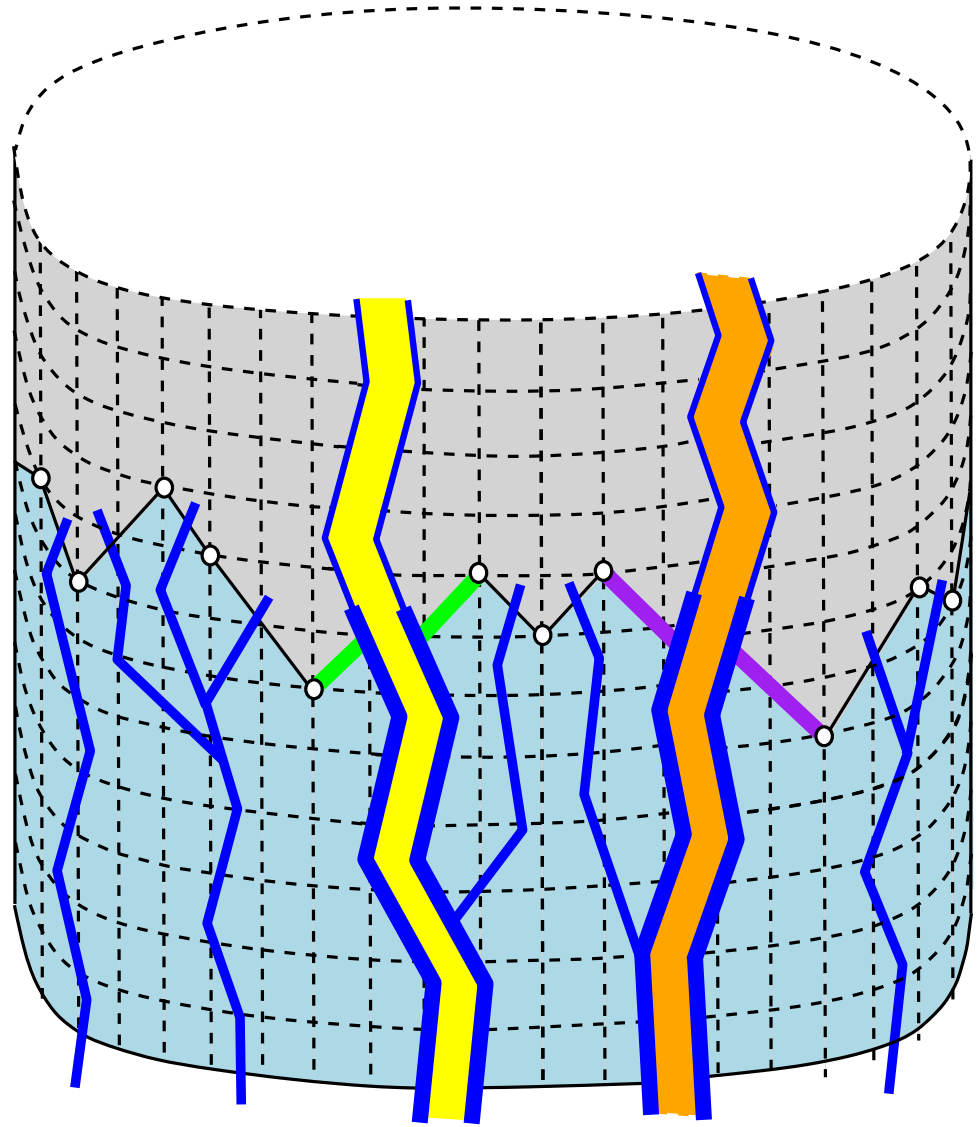
slopes +1 or -1



Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

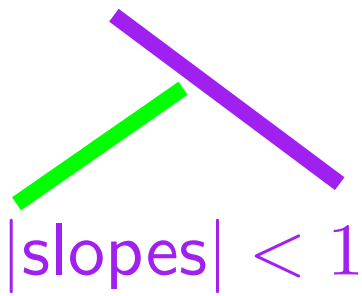
incremental drawing



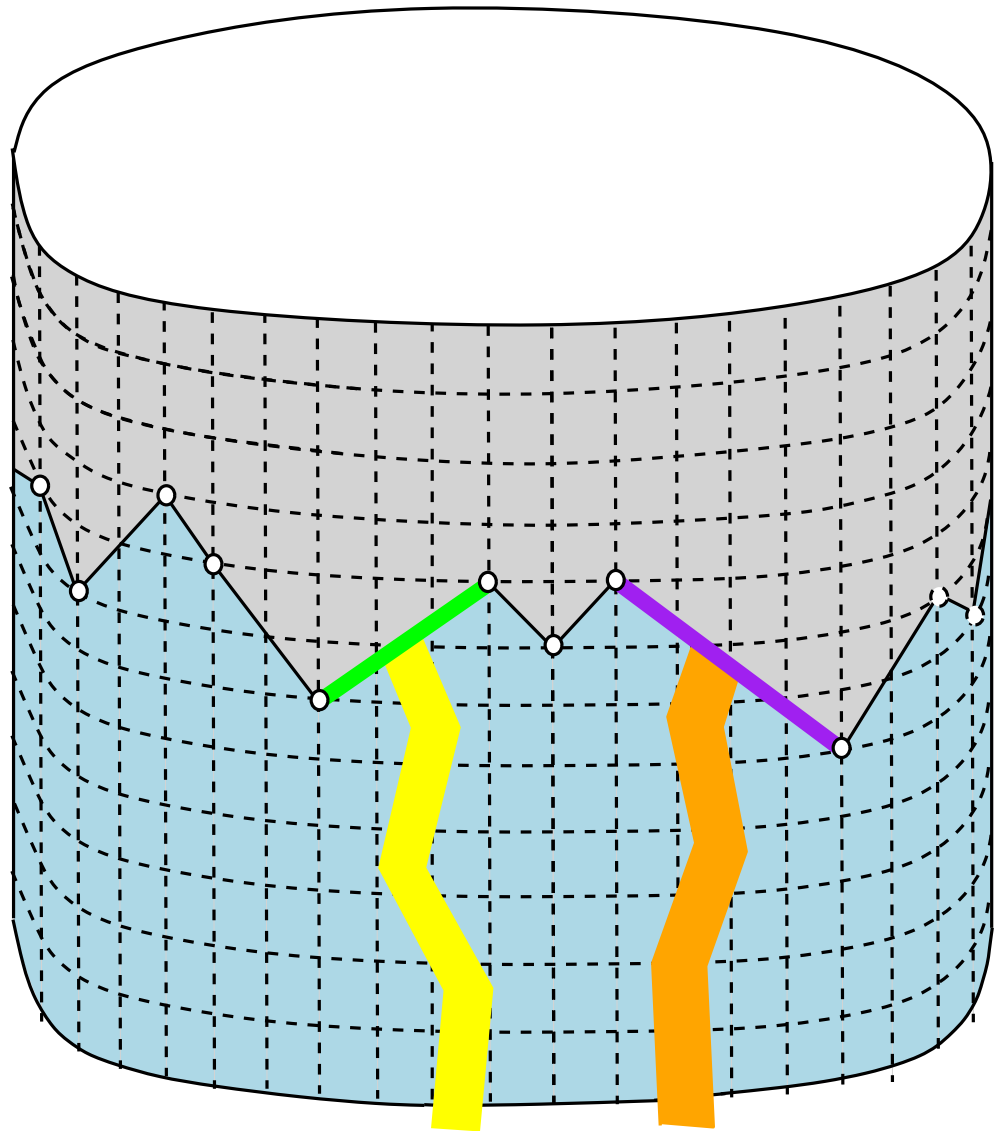
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

incremental drawing



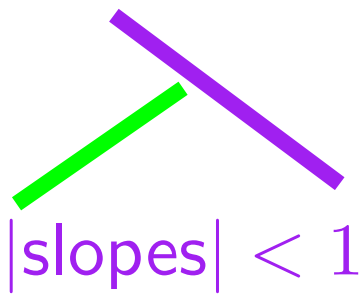
$|\text{slopes}| < 1$



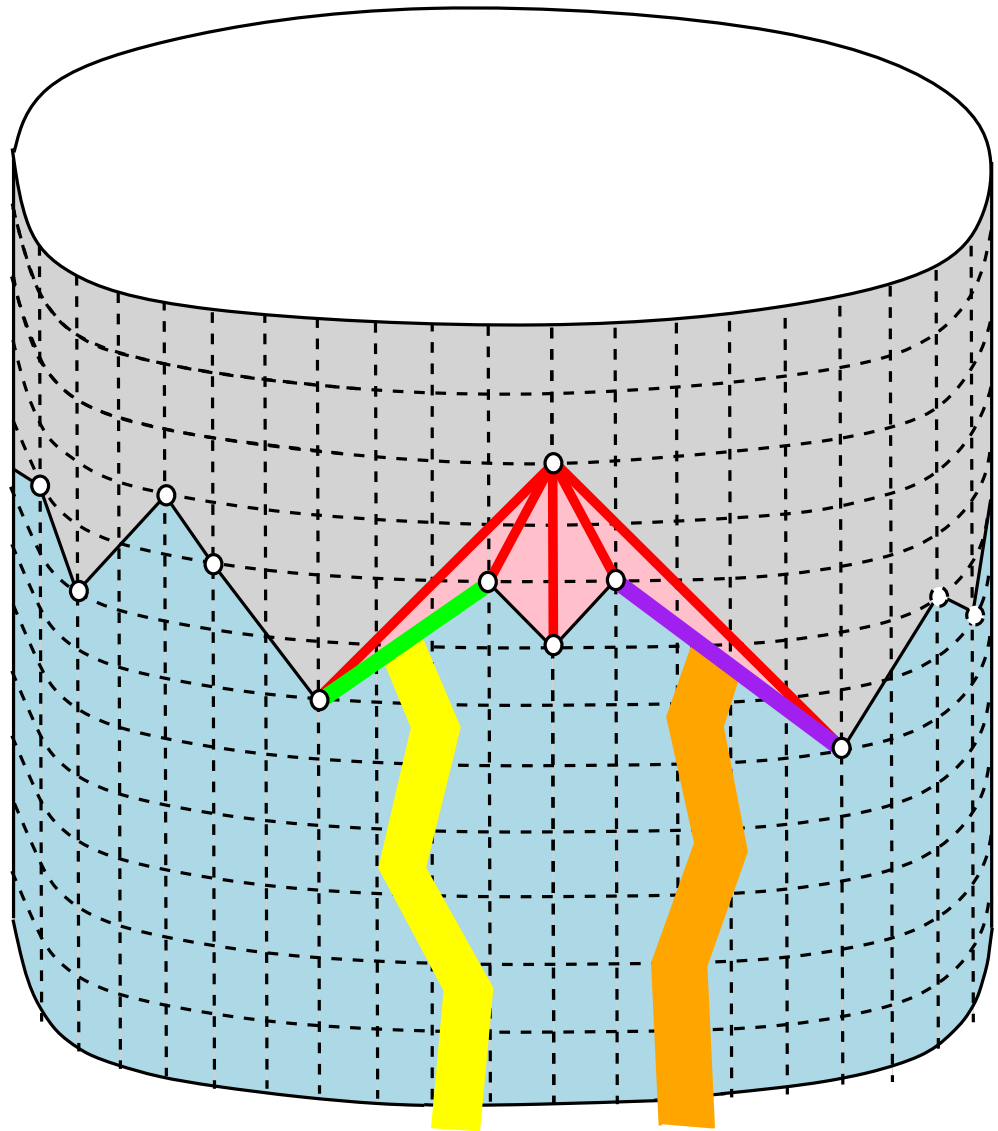
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

incremental drawing



$|\text{slopes}| < 1$

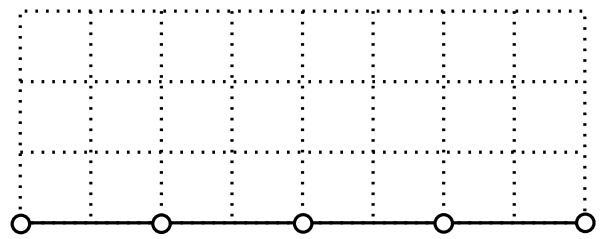


Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

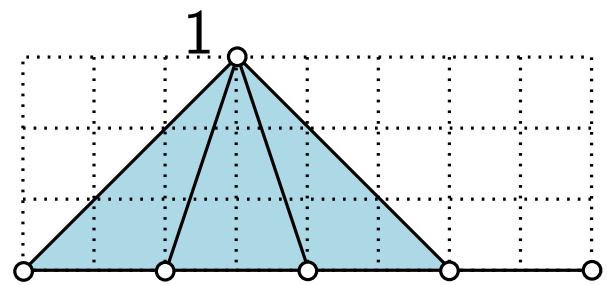
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



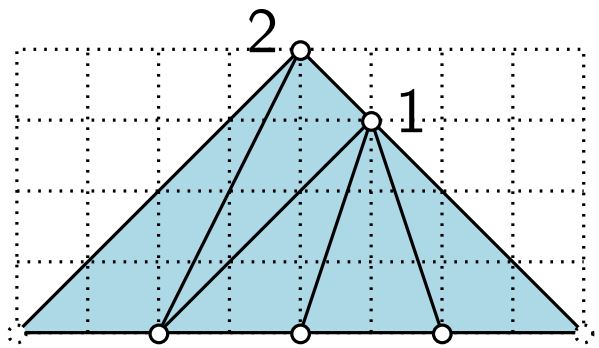
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



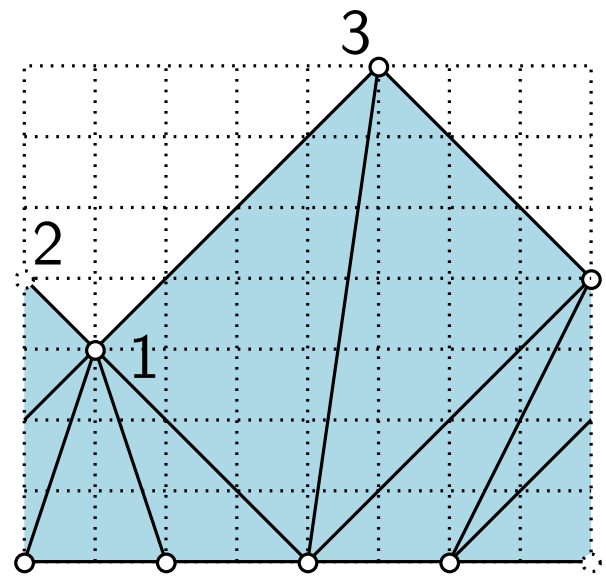
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



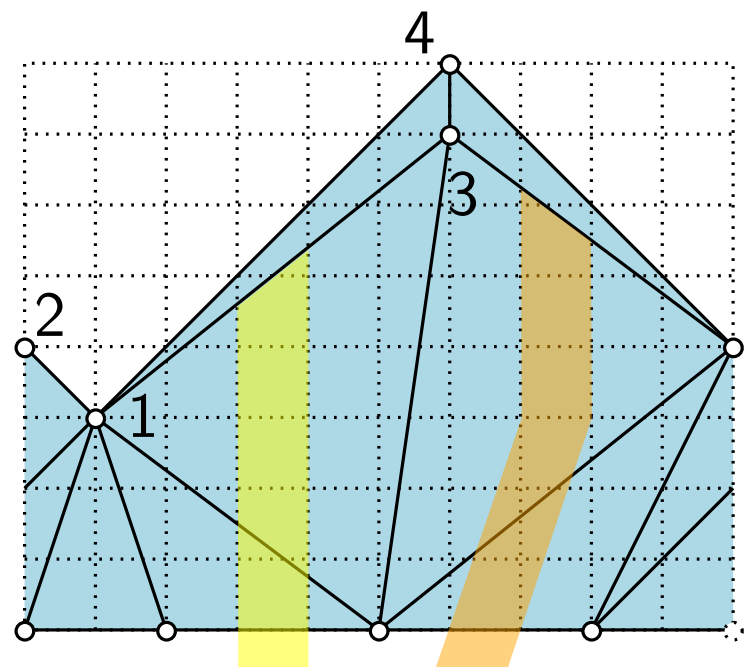
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



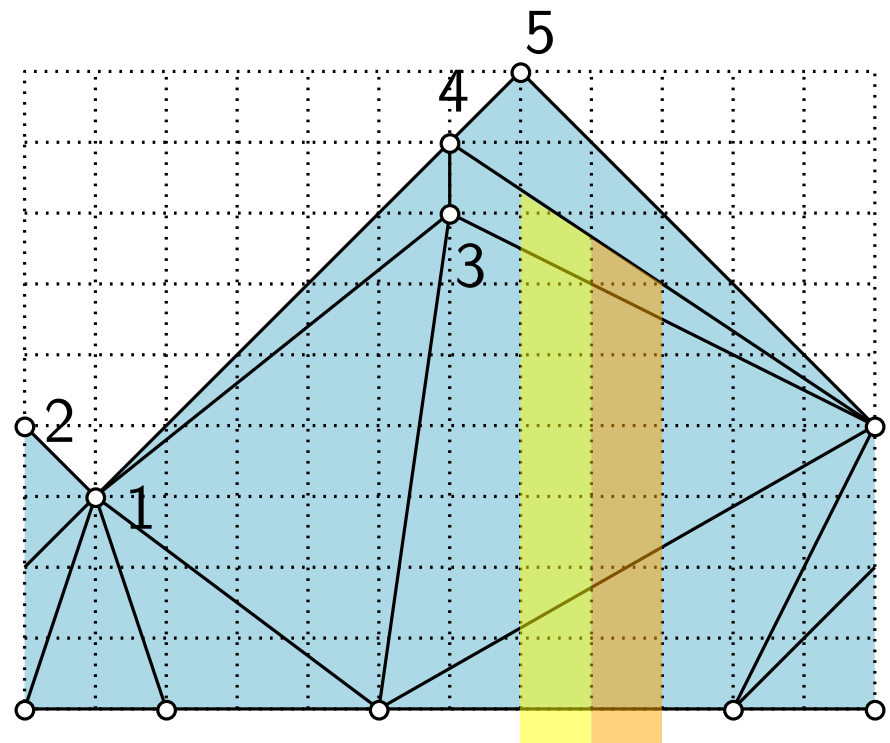
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



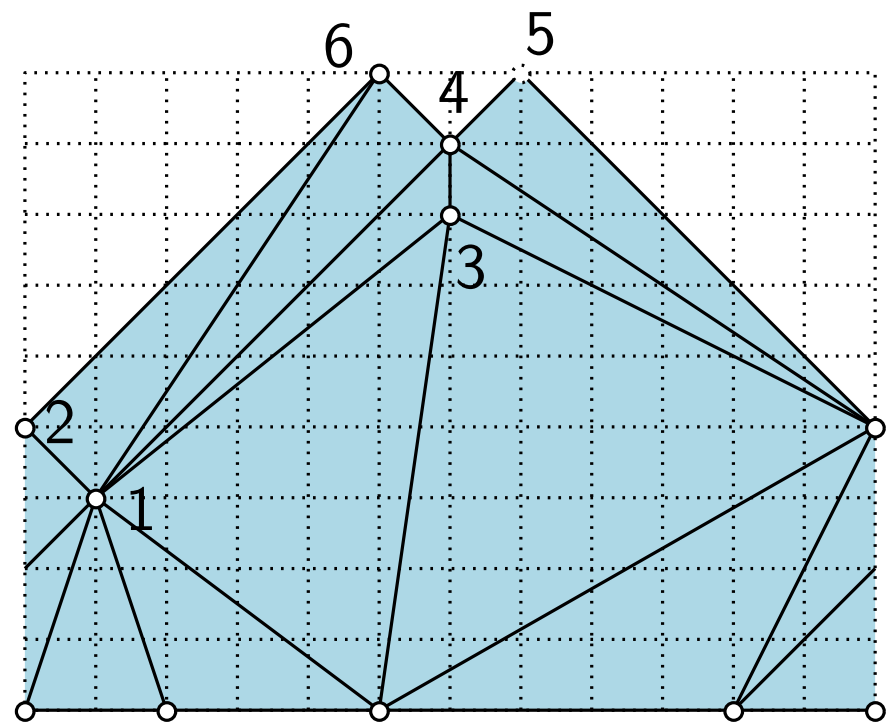
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



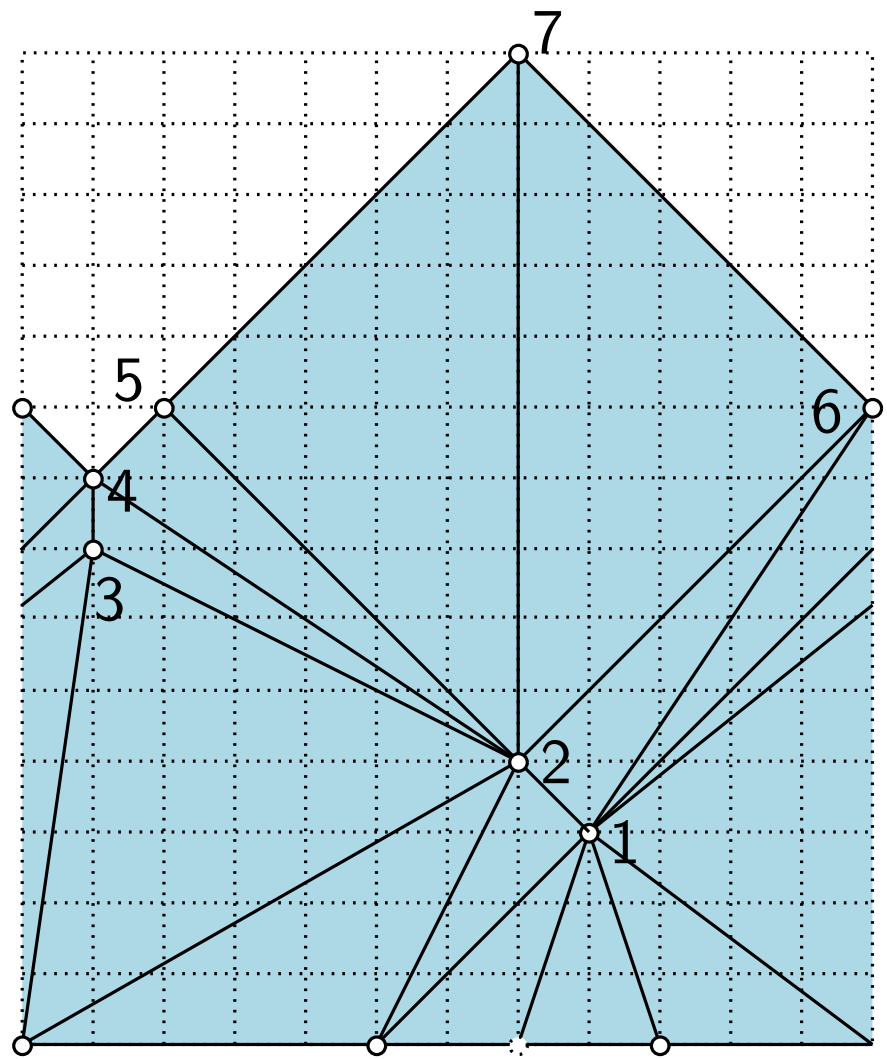
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



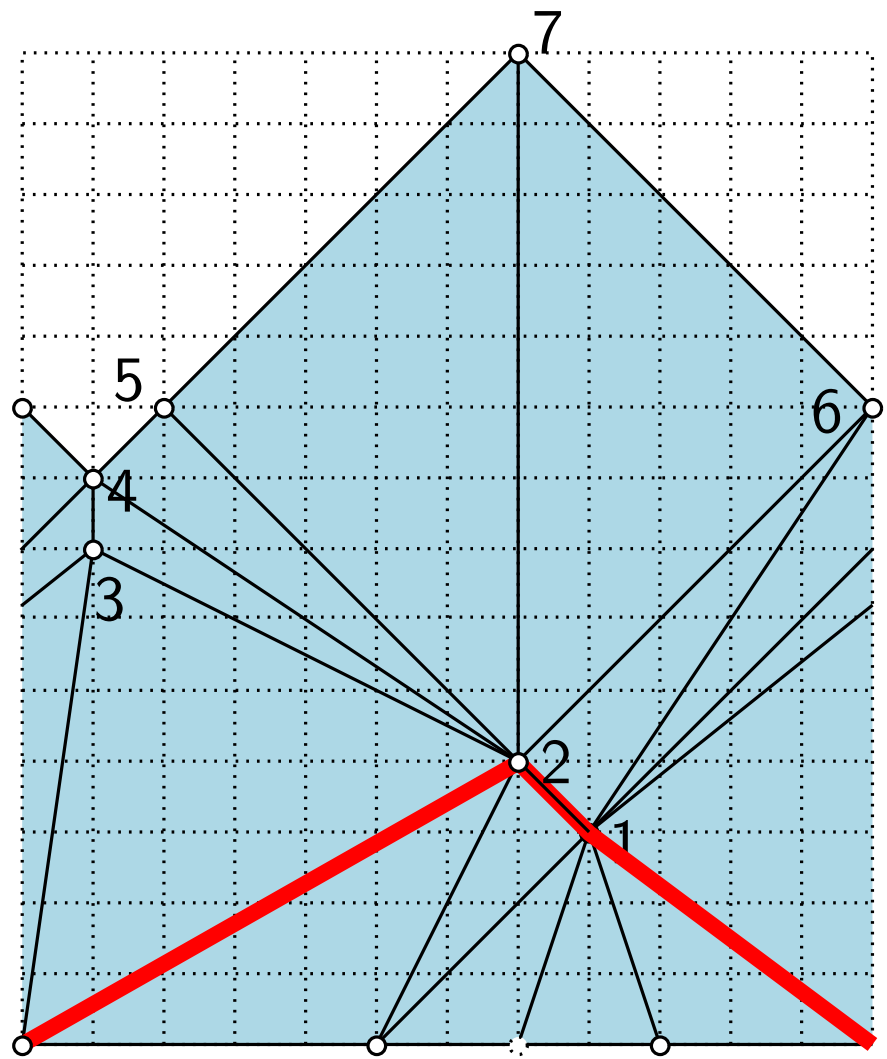
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



Algorithm, incremental drawing

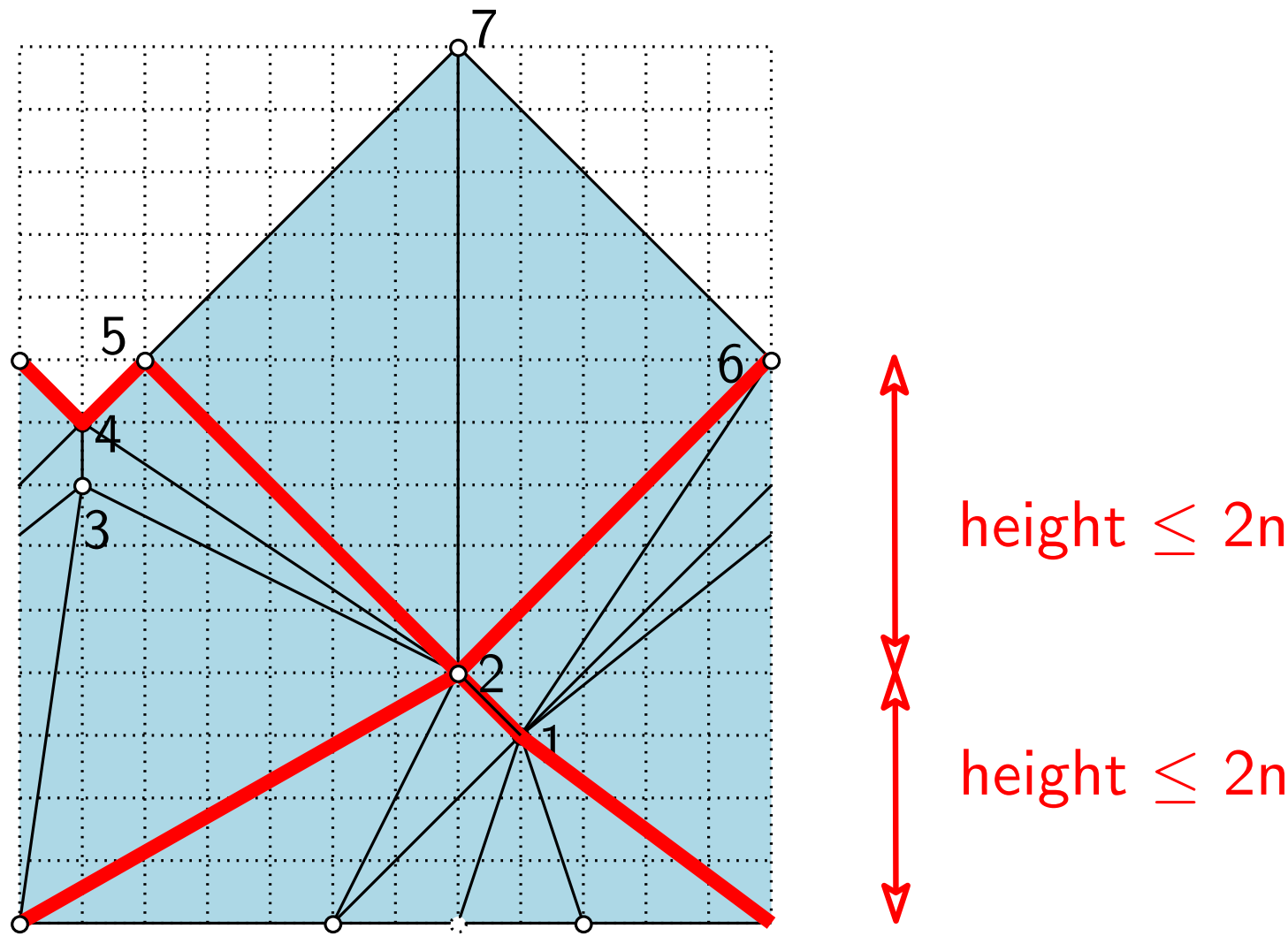
triangulation of a cylinder, no chords, no 2-cycles



height $\leq 2n$

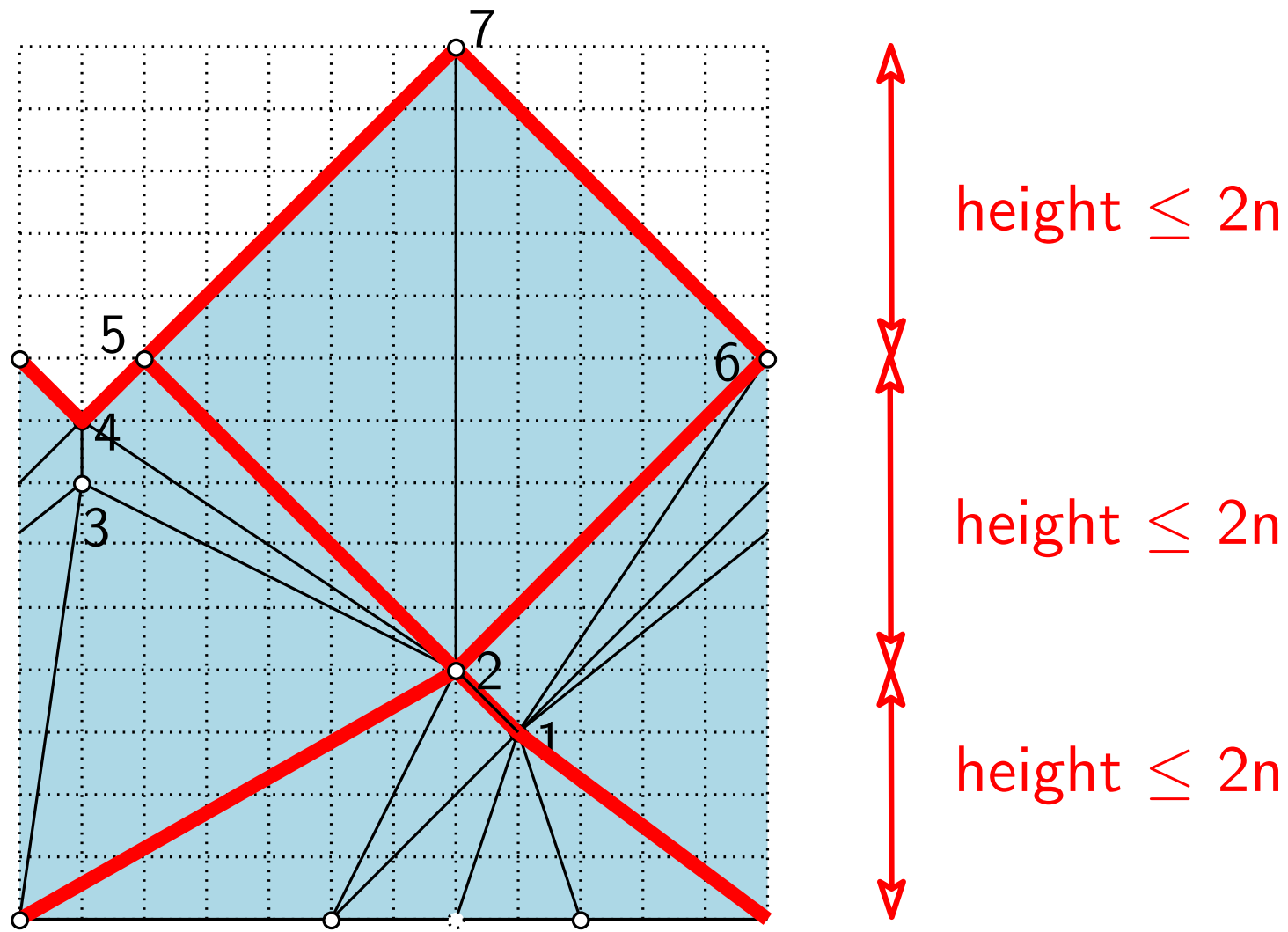
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



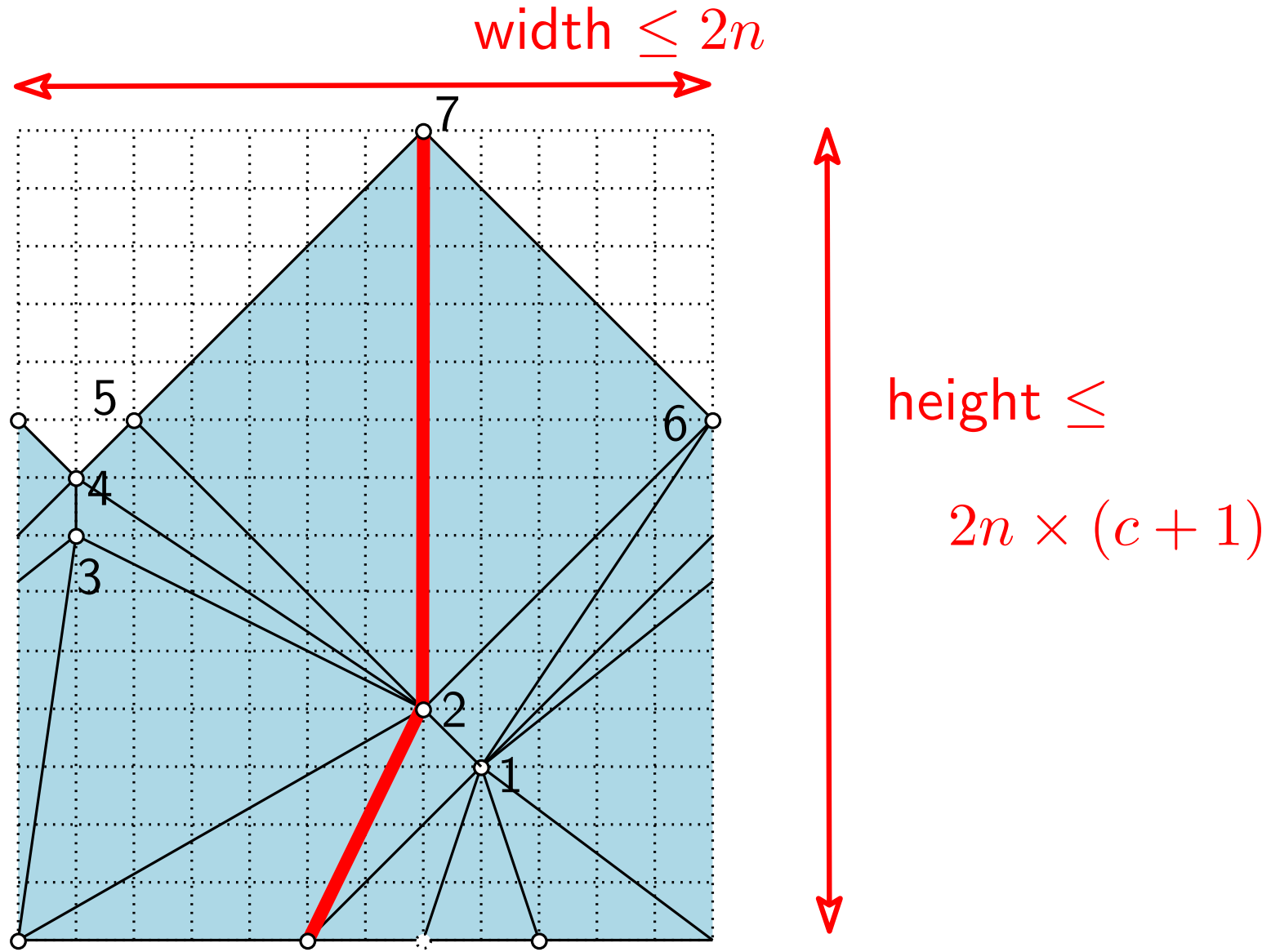
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles



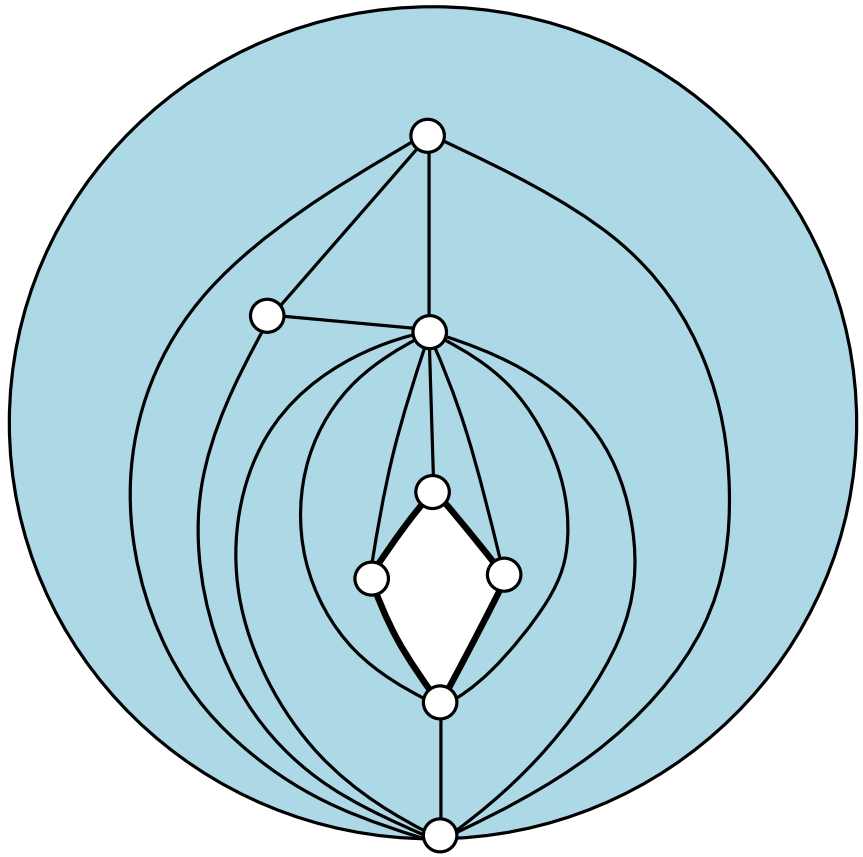
Algorithm, incremental drawing

triangulation of a cylinder, no chords, no 2-cycles

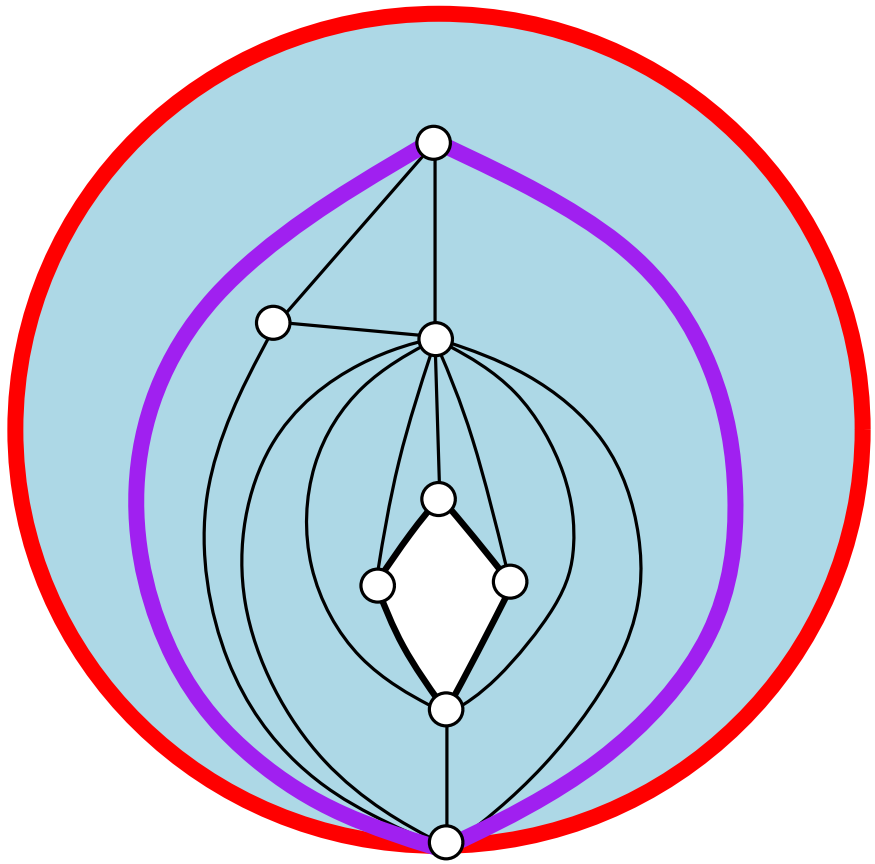


Algorithm, 2-cycles and loops

Algorithm, 2-cycles and loops



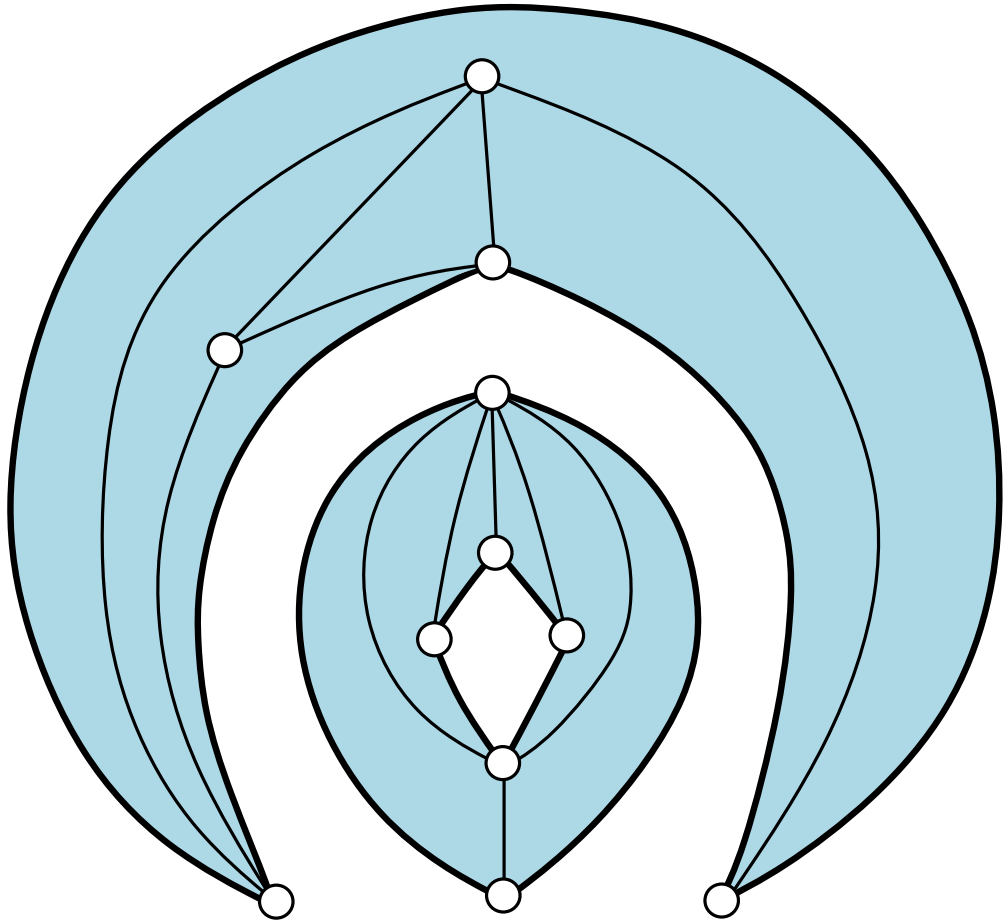
Algorithm, 2-cycles and loops



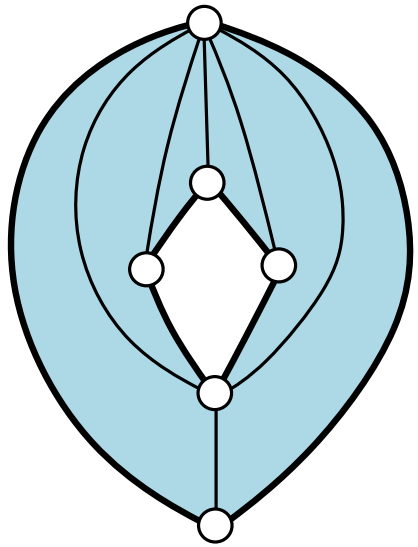
loop

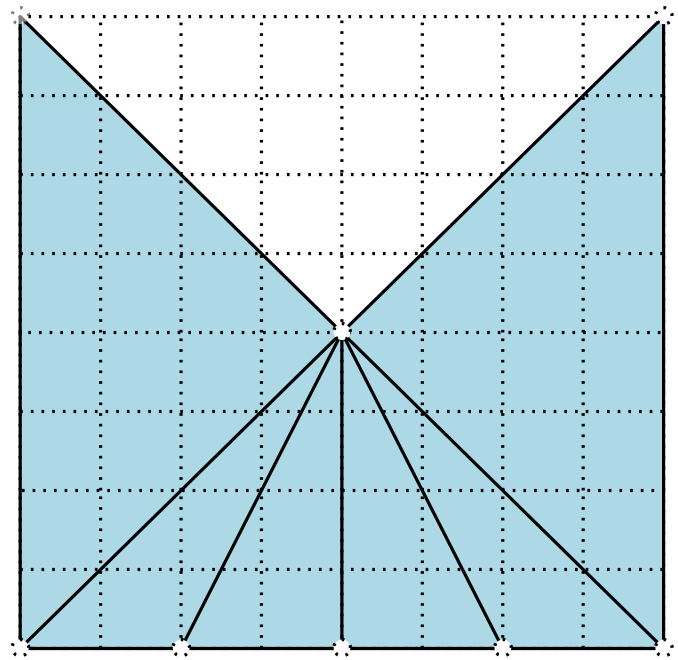
2-cycle

Algorithm, 2-cycles and loops

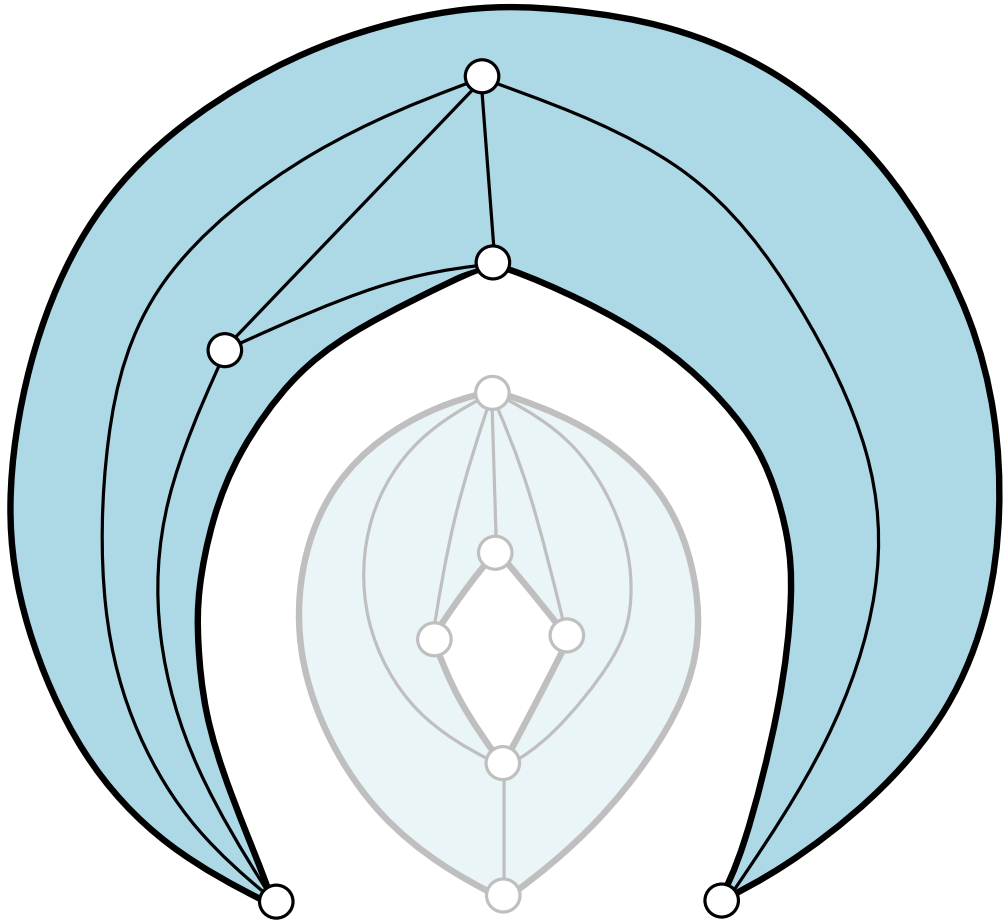


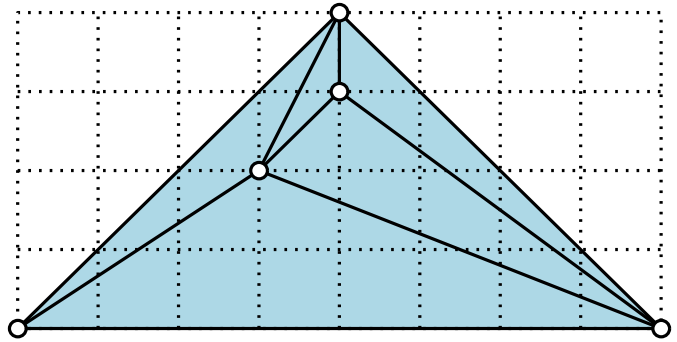
Algorithm, 2-cycles and loops

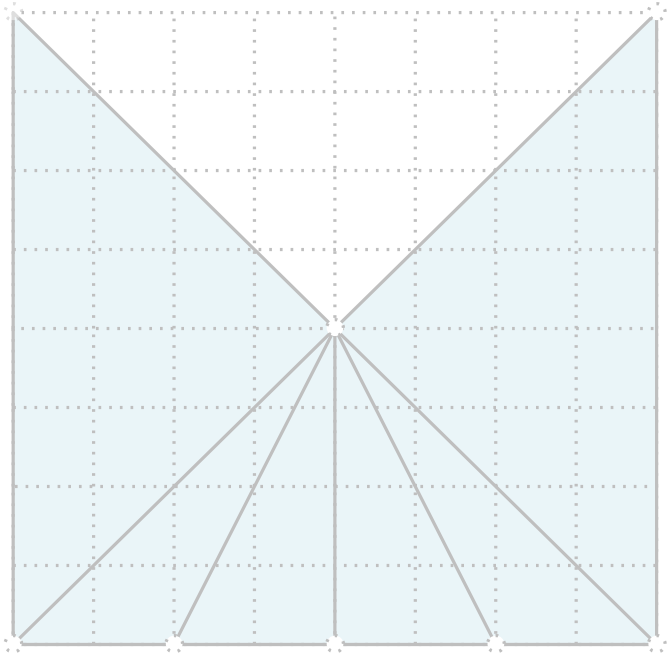




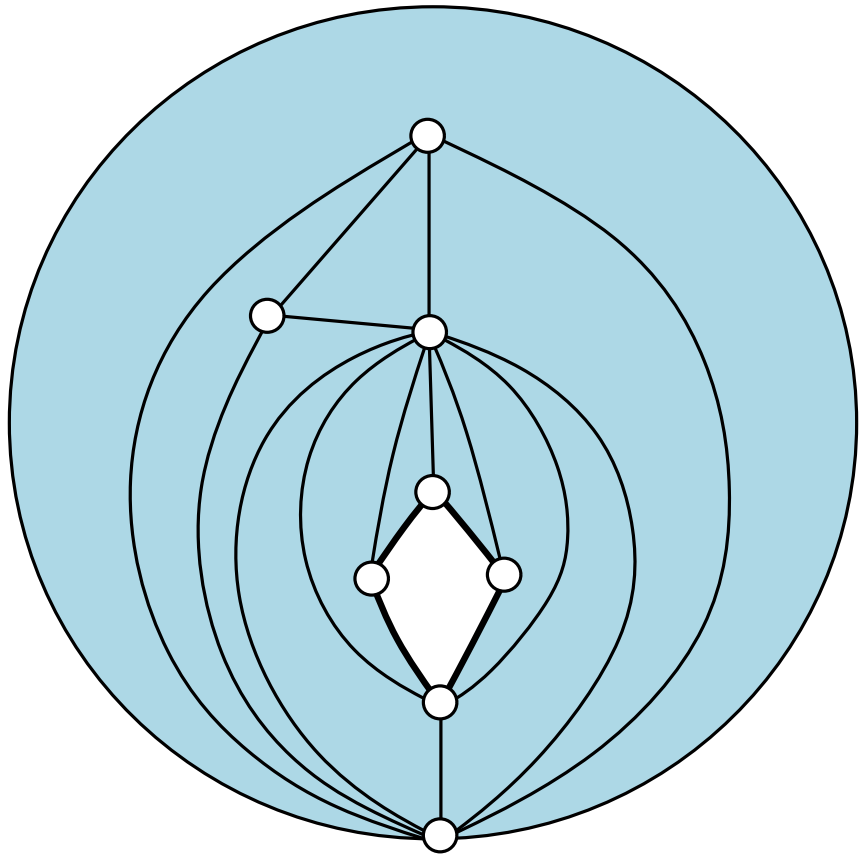
Algorithm, 2-cycles and loops

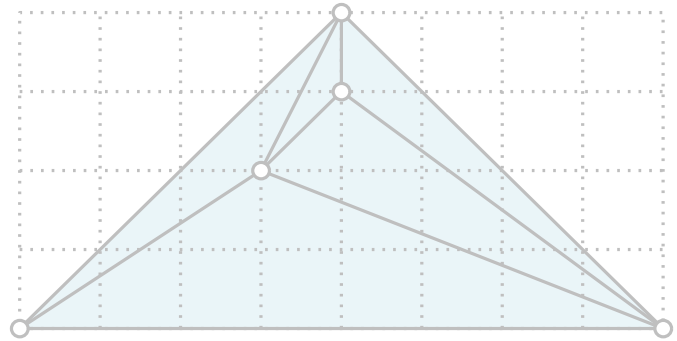


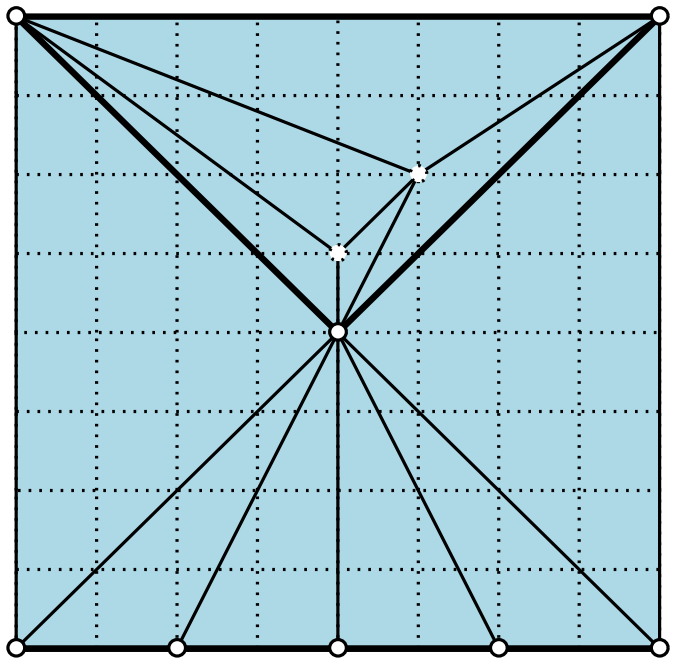




Algorithm, 2-cycles and loops

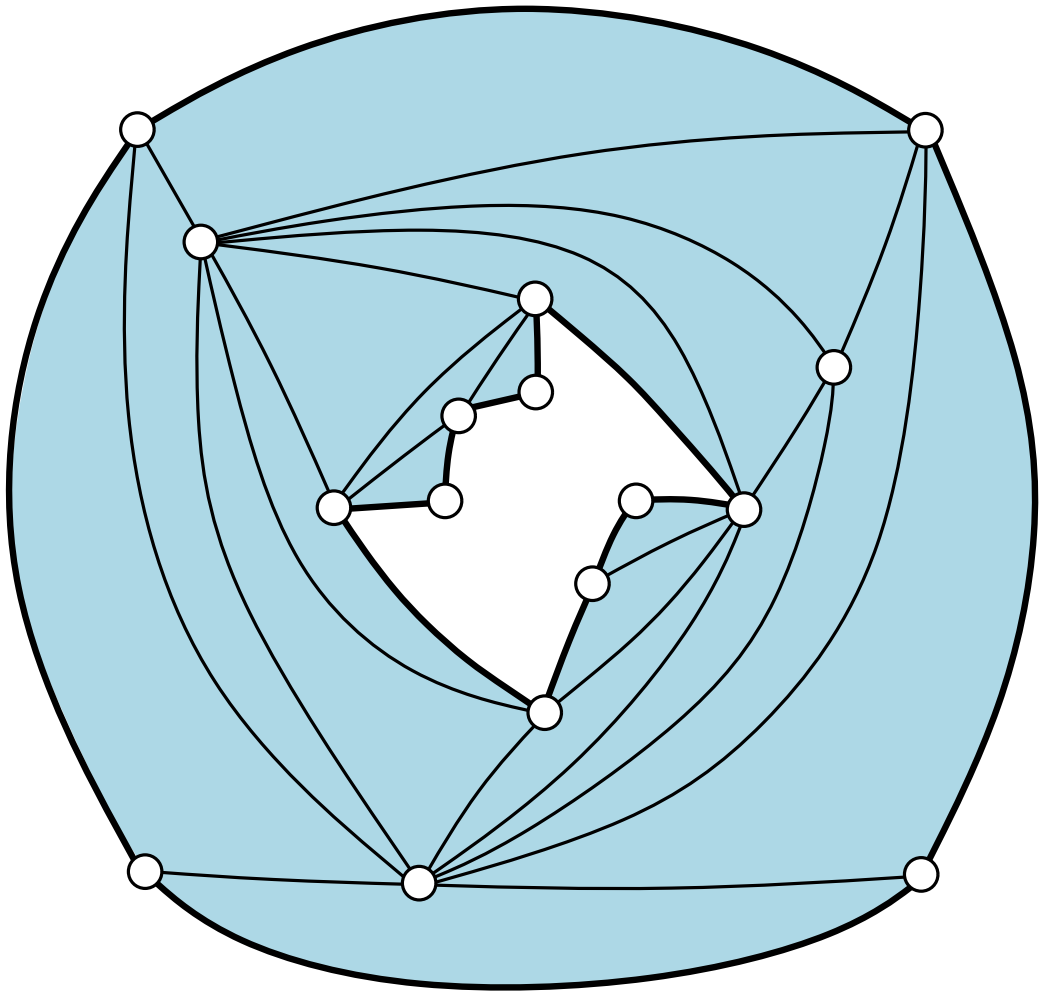




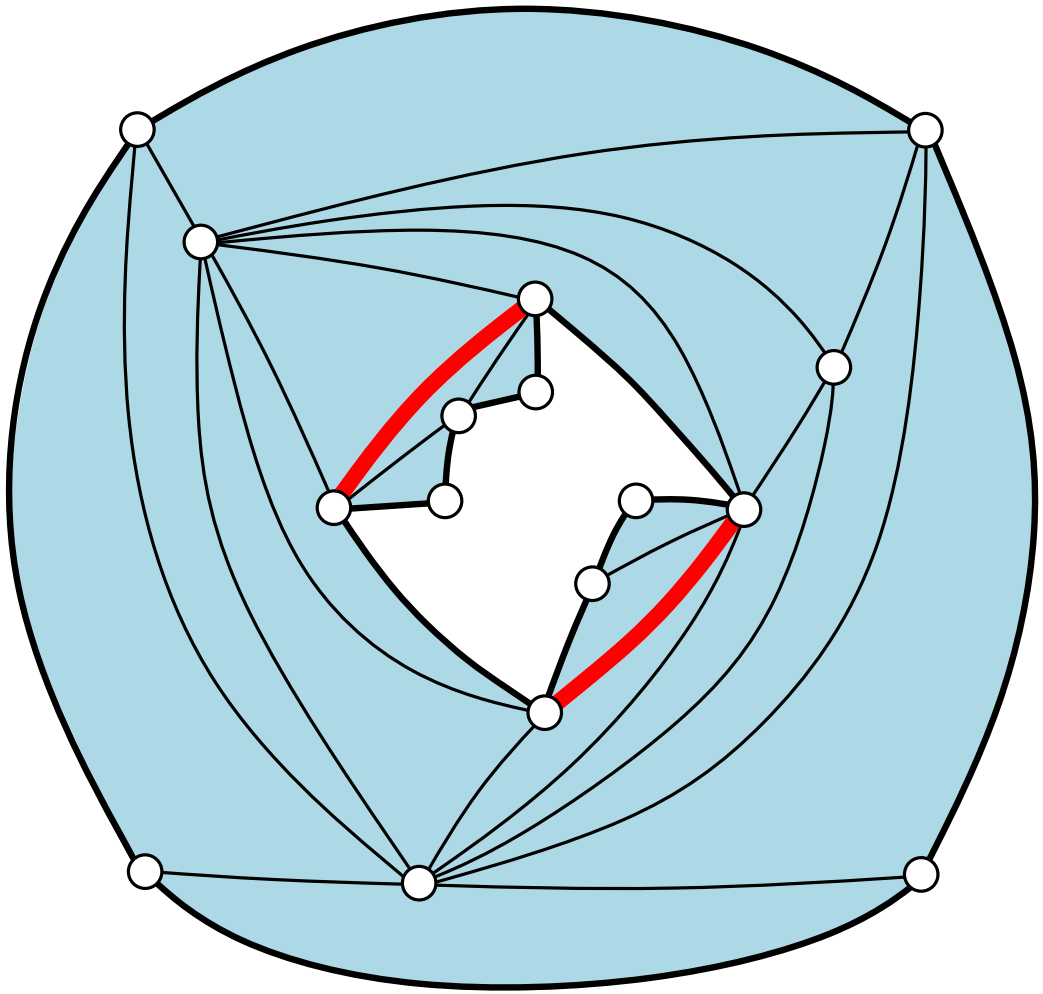


Algorithm, chords

Algorithm, chords

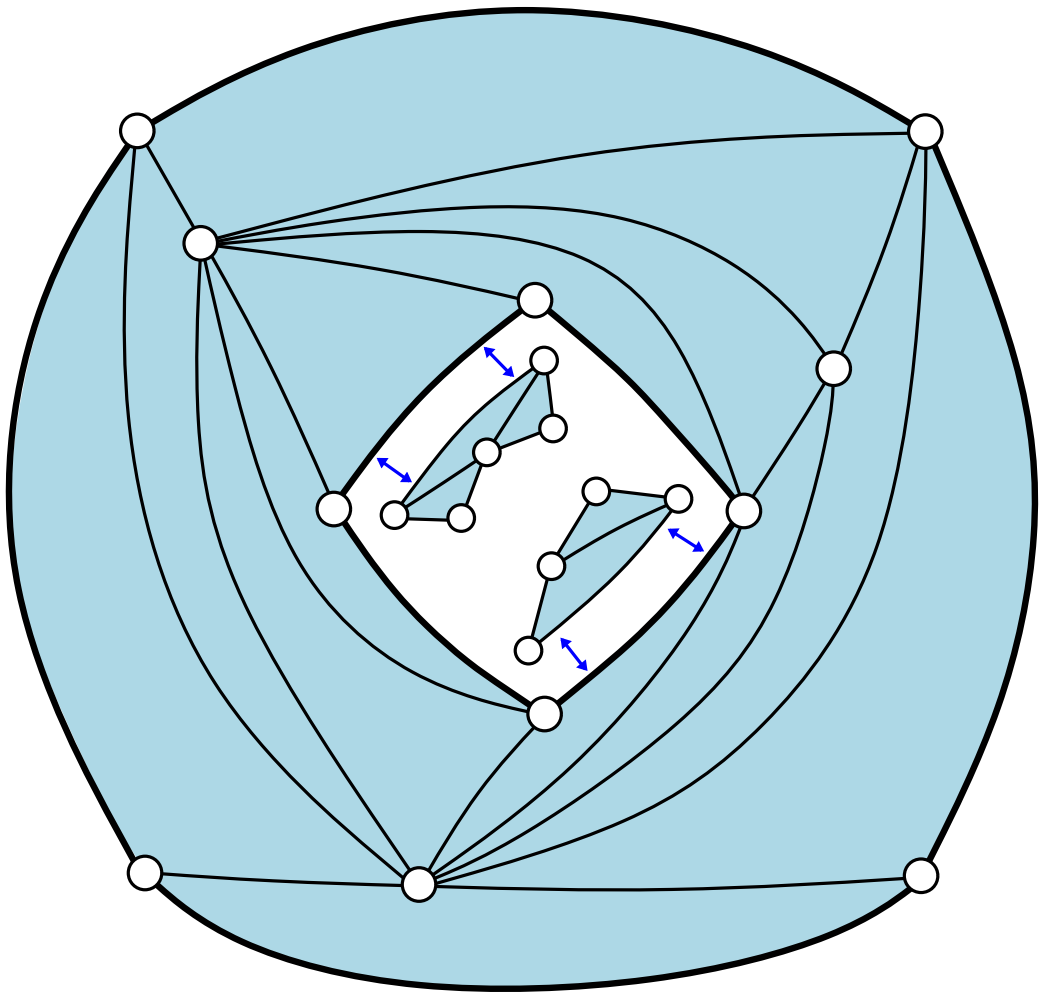


Algorithm, chords

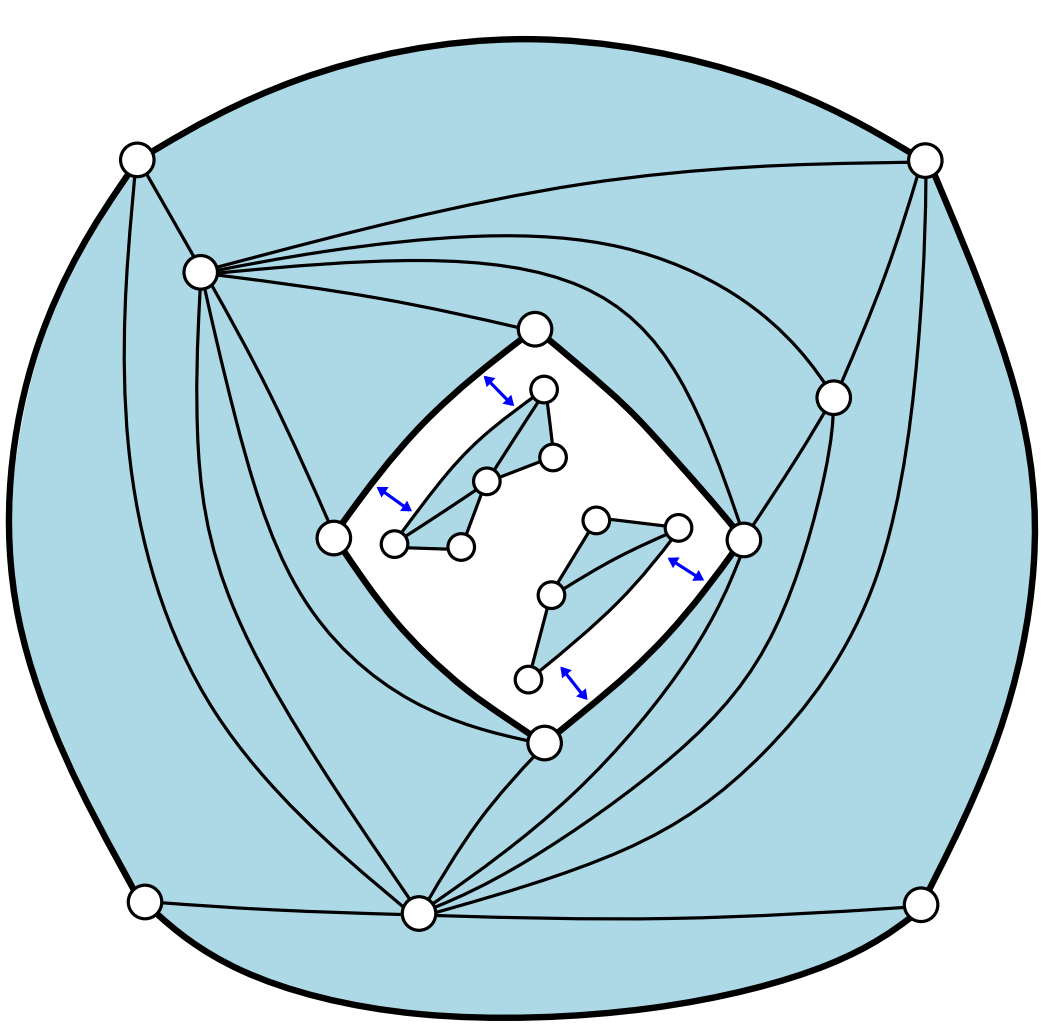


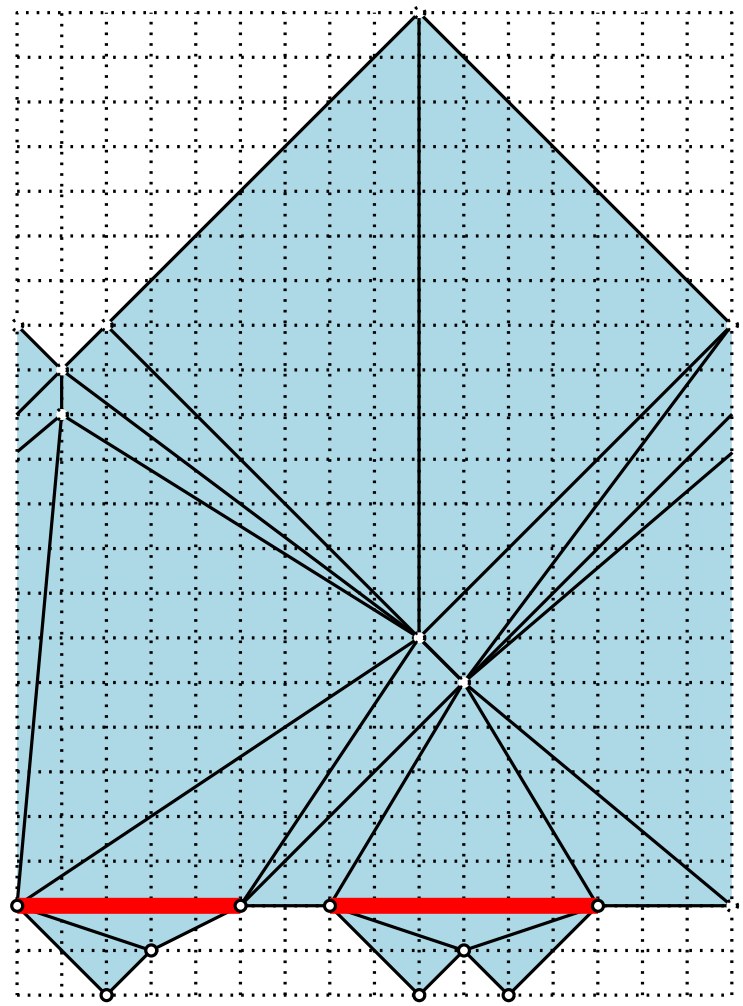
chords

Algorithm, chords



Algorithm, chords

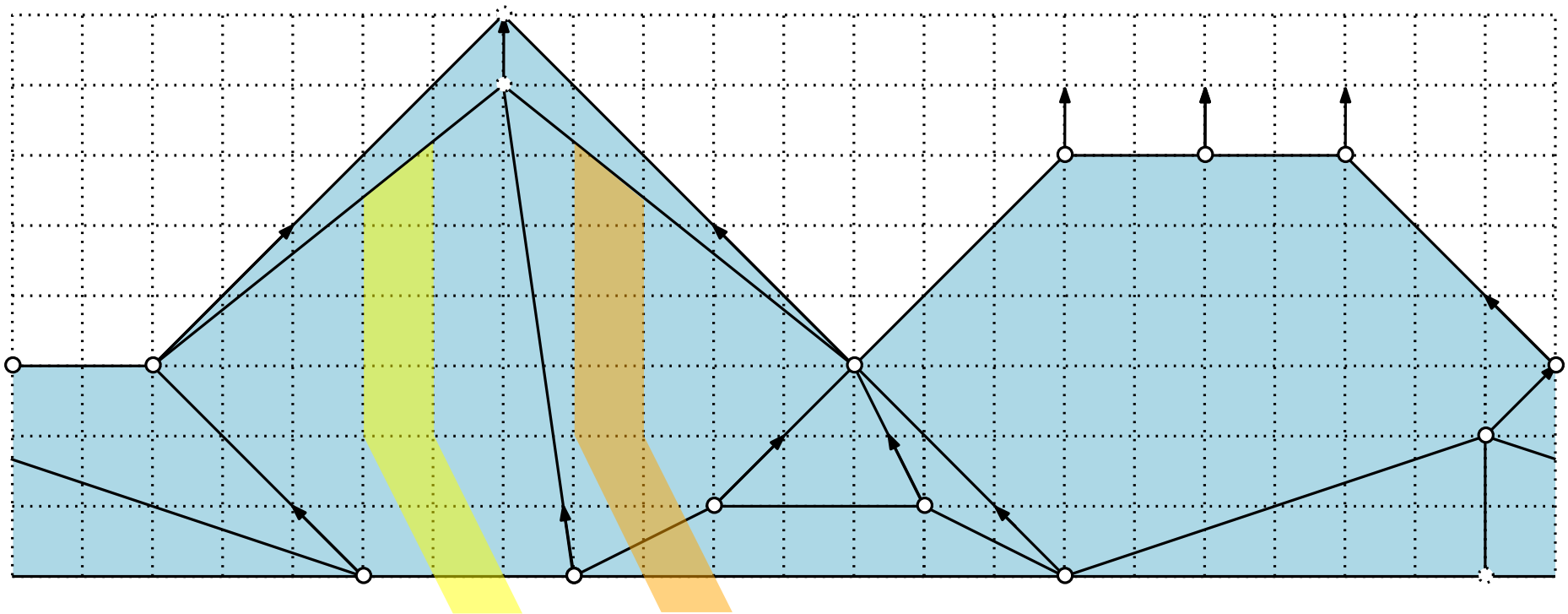




Algorithm, maps

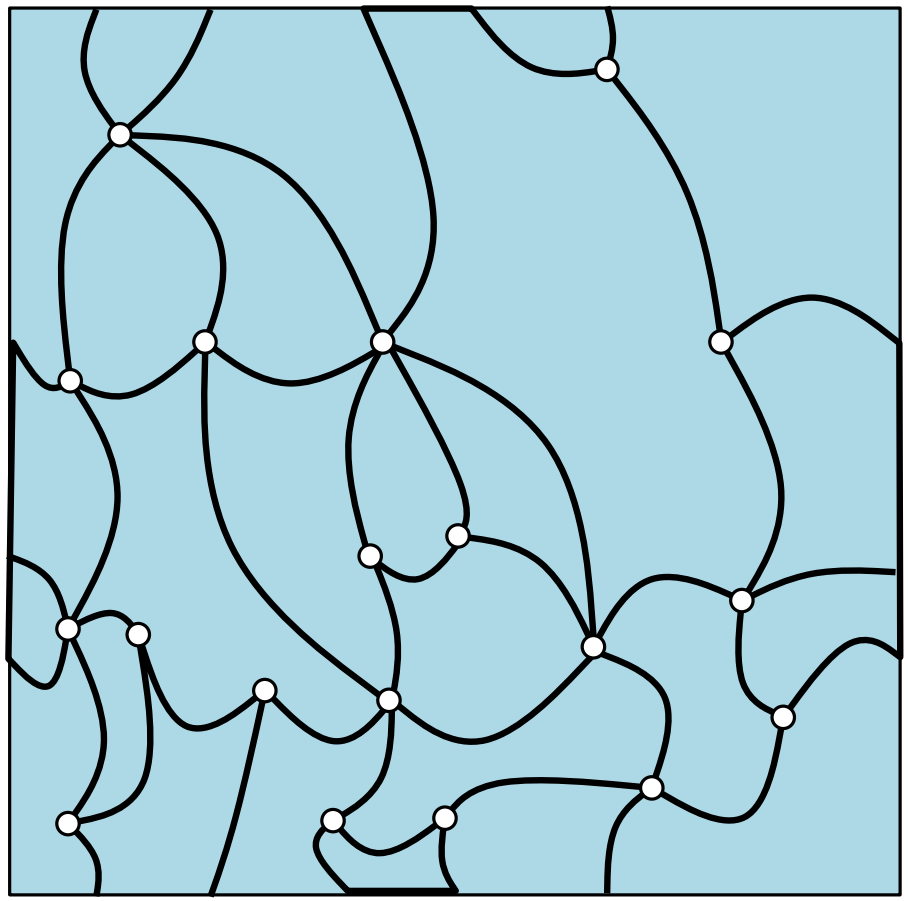
several adaptations

slopes +1 or -1 or 0

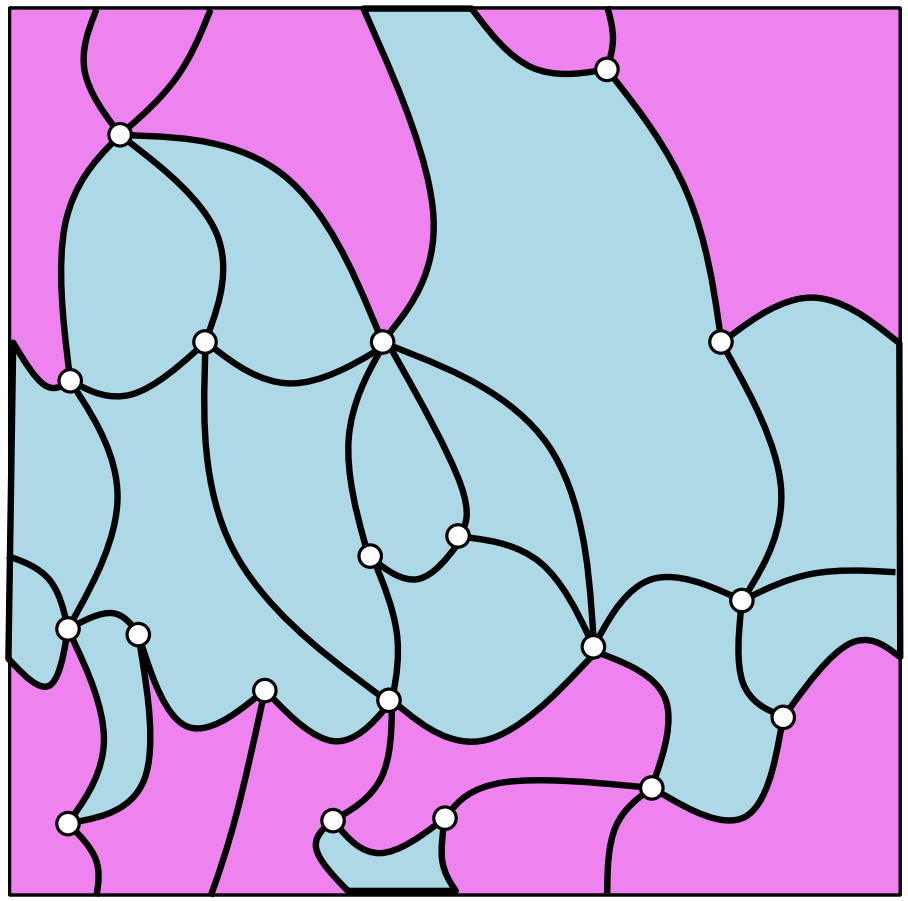


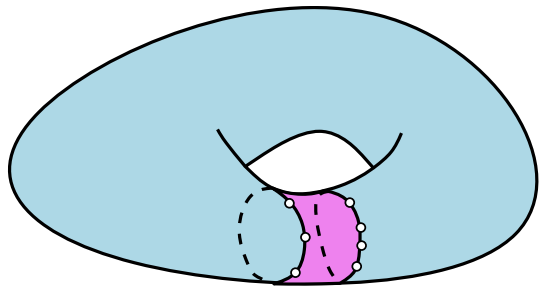
Algorithm, from cylinder to torus

Algorithm, from cylinder to torus



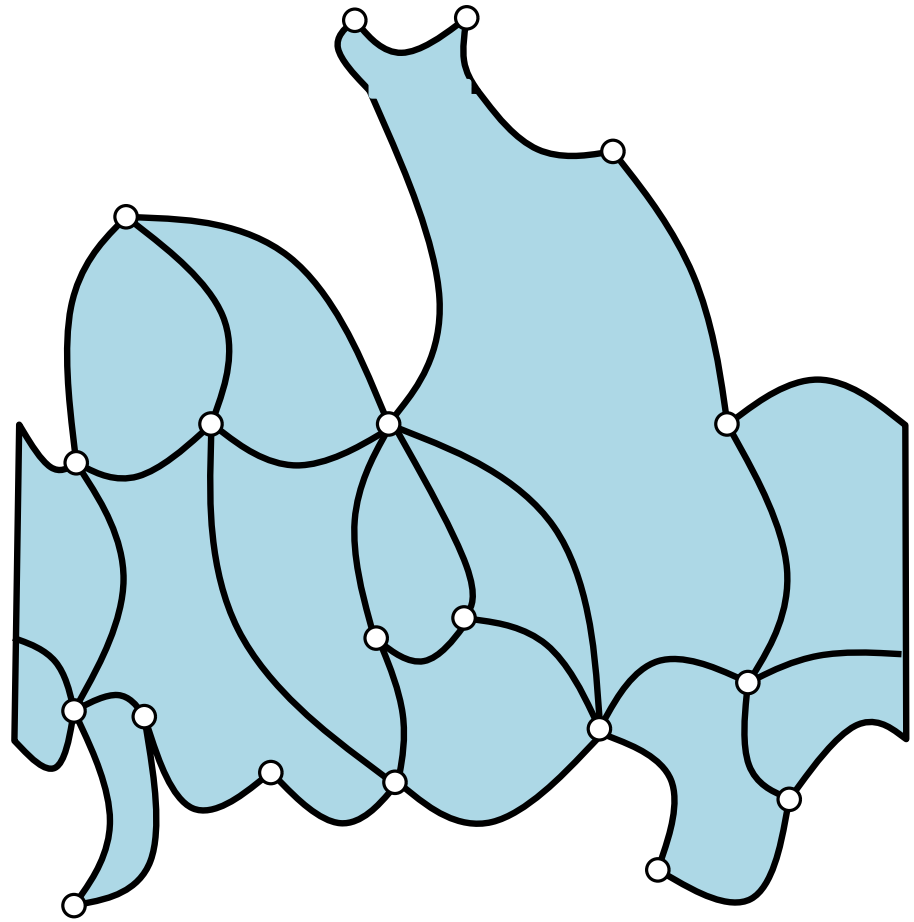
Algorithm, from cylinder to torus

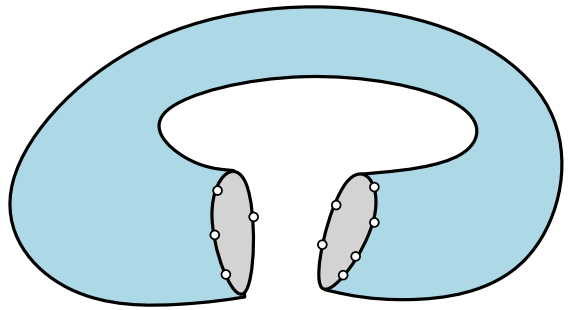




tambourine

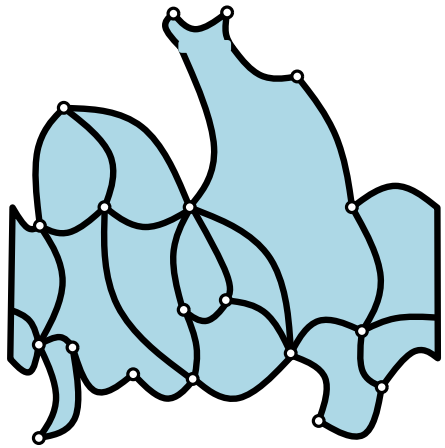
Algorithm, from cylinder to torus



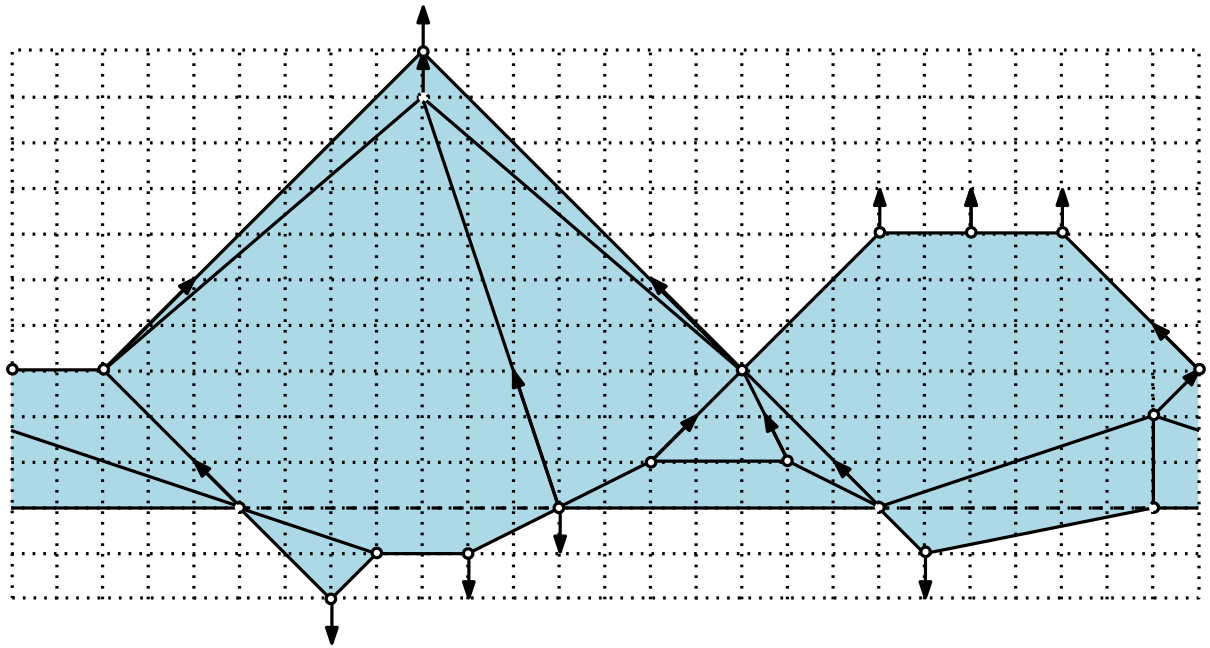


cylinder

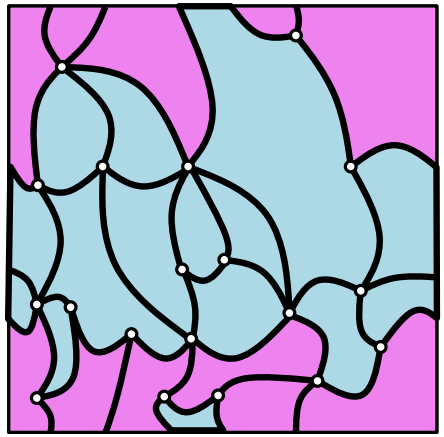
Algorithm, from cylinder to torus

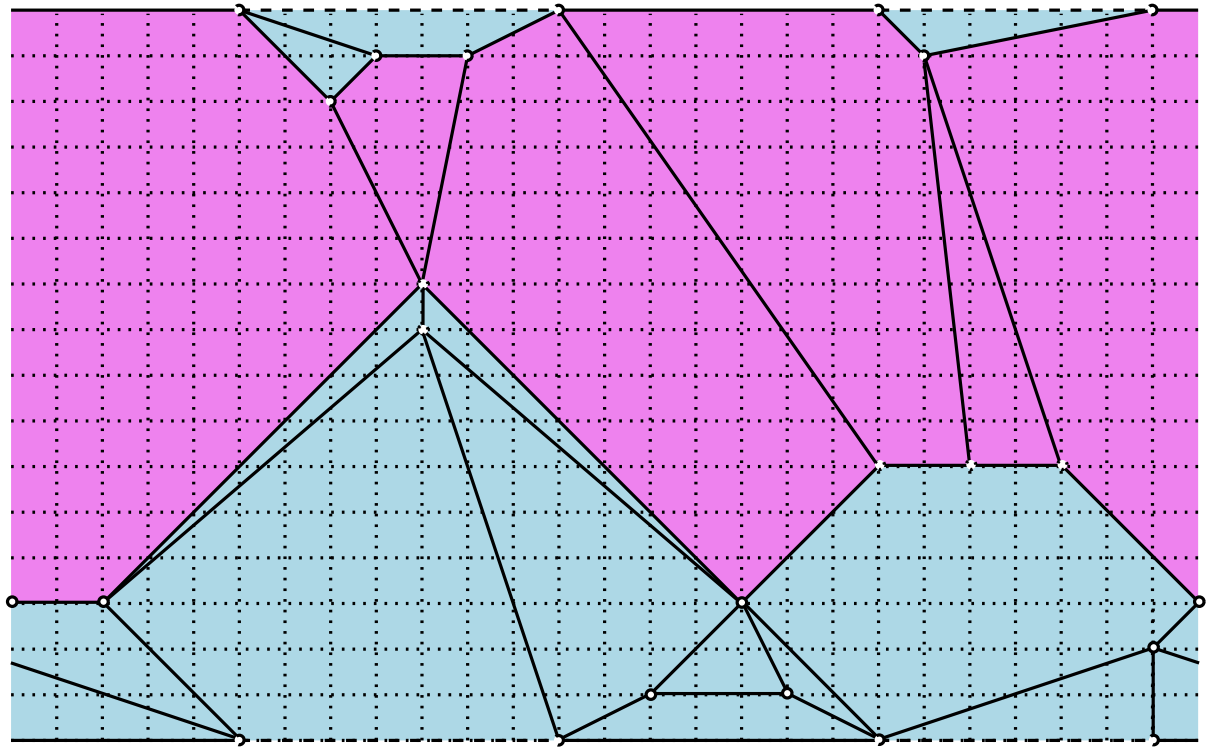


cylinder



Algorithm, from cylinder to torus



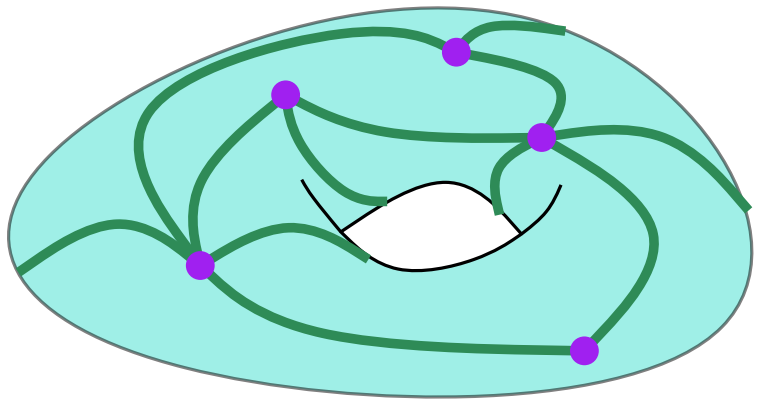


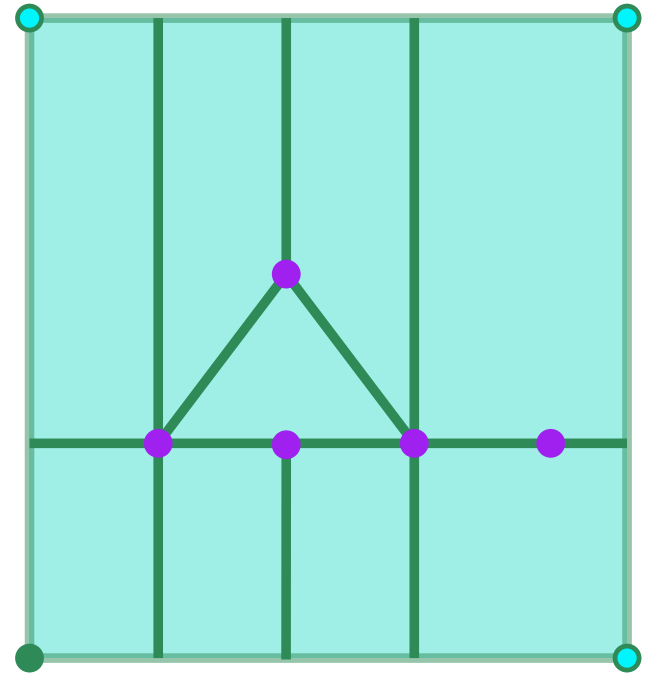
torus

Given a map on a torus

Given a map on a torus

(essentially 3-connected)





Get a

Convex straight line drawing

on grid $2n \times (1 + 2n(c+1))$

$$c \leq \sqrt{2n}$$